Life-Cycle Patterns of Earnings Shocks: A Bayesian Approach

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January, 2024

Abstract

In this paper, I introduce a novel approach to analyzing life-cycle patterns in earnings shocks by applying a Bayesian Logistic Smooth Transition Autoregressive (LSTAR)(1) model, characterized by its rich heterogeneity, to the Panel Study of Income Dynamics data. A key strength of this methodology is its capacity to dissect the interplay among various determinants, such as age, job tenure, income levels, and assets, which are often overlooked in conventional income dynamics models. This study specifically focuses on age and job tenure, offering a comparative analysis with traditional estimates derived from standard models. I find that individuals under 29 experience earnings shocks with higher volatility and reduced persistence compared to their older counterparts. Furthermore, the analysis reveals that longer job tenure is associated with improved resilience to shocks, thereby enhancing our understanding of the underlying economic mechanisms influencing income dynamics. The findings underscore the critical role of heterogeneity in income dynamics models and their potential implications for the calibration of macroeconomic models.

Key words: Income Processes, Nonlinear Time-Series Models, Bayesian Estimation.

JEL Classifications: C01, C11, C15, C33, D31, E27, and J31.

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1. Introduction

Over the last four decades, the labor market has experienced significant transformations that are yet to be fully integrated into our macroeconomic models' calibration of income dynamics. The challenge lies in that incorporating the complex nature of income dynamics into a general dynamic equilibrium model is not trivial, leading to a disconnect between theoretical representations of income shocks in the model and the empirical observations. For instance, the widely acknowledged fact that income shocks affect workers at different career stages differently has not been systematically factored into the standard model calibrations of income shocks. This oversight, among others, raises crucial questions about the potential impact of life-cycle patterns on labor earning risk and their implications for macroeconomic analysis. If these patterns do exist and diverge from conventional model predictions, they could fundamentally reshape our understanding of key economic outcomes such as consumption-saving decisions, wealth inequality, insurance mechanisms, and the effects of business cycles and public policies.

This paper addresses several pivotal questions: Are there discernible life-cycle patterns in idiosyncratic earnings shocks? How does aging influence these shocks? What other factors contribute to the complexity of earnings dynamics? And to what extent do earnings shocks correlate with job tenure? More broadly, this research probes whether current macroeconomic models might be overlooking critical aspects of income dynamics, necessitating reevaluating how income shocks are calibrated.

To explore these questions, this study introduces a novel approach by employing a Bayesian Logistic Smooth Transition Autoregressive (LSTAR)(1) model to analyze labor income dynamics within the Panel Study of Income Dynamics (PSID) from 1968 to 2019. The LSTAR(1) model is distinguished by its incorporation of significant heterogeneity in innovations, moving beyond traditional linear AR structures to embrace a nonlinear framework that captures the intricate nature of income changes across an individual's life span.

A vital feature of this research is its examination of various determinants of earnings shocks, such as age, job tenure, income levels, and asset accumulation, which are often overlooked in standard models. The study specifically investigates the impact of age and job tenure on earnings shocks, providing a comparative analysis with traditional estimates and highlighting the potential for extending the analysis to other dimensions. By expanding the analysis beyond age, this paper challenges the prevailing simplifications in macroeconomic models, which tend to overlook the age-dependent variability of income shocks. This research emphasizes the need for more sophisticated models, such as the LSTAR(1), to capture the heterogeneity and complexity inherent in income dynamics accurately.

Additionally, this paper contributes to the emerging field of Bayesian analysis in income dynamics, offering a methodologically innovative approach with broad applicability and potential for yielding deeper insights. Adopting a Smooth Transition Autoregressive (STAR) model within a Bayesian framework represents a significant methodological advancement, providing a versatile tool for examining shock nonlinearities and their macroeconomic implications. My findings can be summarized as follows.

First, there is a discernible life-cycle pattern in idiosyncratic earnings shocks. Workers younger than 29 experience shocks with higher variance and a notable likelihood of lower persistence compared to their older counterparts, with a definitive transition in the shock regime occurring assuredly before age 35. The stationary variance of the persistent component of the earnings shocks for younger workers is approximately 1.3 times higher than that for older workers. This transition varies among educational groups, occurring around age 45 for college graduates and age 55 for high school graduates. The variance of shocks over the life cycle also exhibits age-dependent changes, with a significant decline in variance for high school graduates post-50 and an increase for college graduates post-30, peaking at 60% higher towards the end of the life cycle.

Second, employing Bayesian methods alongside traditional GMM estimates reveals comparable results for the AR process's correlation coefficient and variance, as well as the variance of temporary shocks within a Restricted Income Profile (RIP) model. However, incorporating nonlinearities in the model slightly reduces the persistence of shocks to a range between 0.96 and 0.97, revealing a higher kurtosis in the innovation of persistent shocks not captured by GMM techniques. Extending the model to accommodate a more flexible error term distribution demonstrates that a significant majority of workers, 79%, encounter shocks with markedly lower variance (0.0003 versus 0.03 in the standard model) and higher persistence (0.99 versus 0.96). Conversely, the remaining time, roughly every five years, they are subjected to shocks with substantially higher variance (0.25 versus 0.03 in the standard model), underscoring the high kurtosis (11) observed in the data.

Third, the analysis underscores the pivotal role of job tenure as a robust variable influencing the magnitude and frequency of earnings shocks, challenging the sole reliance on age as a determinant. The educational divide in shock experiences among college and high school graduates diminishes when job tenure is considered, suggesting tenure's significant impact on shock resilience. Workers with longer tenure exhibit reduced shock volatility, indicating increased job stability and greater predictability in earnings over time. This tenure-centric perspective harmonizes previously noted educational disparities, advocating for a more comprehensive approach to modeling income dynamics.

Finally, this study reveals that conventional models fail to capture the full complexity of income dynamics. The proposed model, characterized by its innovative framework where the income process is articulated as a convex combination of two Autoregressive (AR) processes, offers a seamless extension to macroeconomic calibrations. This model's structure lends itself to straightforward approximation through a discrete Markov process, as elucidated by Vandekerkhove (2005), ensuring its tractability. Consequently, this facilitates the application of standard calibration techniques, thereby enhancing the model's utility in macroeconomic analyses and policy formulations.

1.1. Literature Review

The empirical exploration of income dynamics has yielded a diverse array of models that capture varying degrees of complexity and heterogeneity. These models are often distinguished by the heterogeneity embedded within the conditional mean and variance of income processes.

A prevailing trend in macroeconomic literature involves categorizing models based on individual earning profile growth rates. Models assuming a uniform growth rate across individuals are termed Restricted Income Profile (RIP) models, while those attributing unique growth rates to each individual are known as Heterogeneous Income Profile (HIP) models. This dichotomy, inspired by human capital theories, suggests that individuals with differing abilities yield varied returns on human capital investments. Notable examples of HIP models devoid of heterogeneity in persistence or conditional variance include studies by Baker (1997), Baker and Solon (2003), and Guvenen (2009), among others. Conversely, RIP models have been exemplified in works by Abowd and Card (1989), Hryshko (2012), and MaCurdy (1982).

Another segment of the literature focuses on modeling the conditional variance of income processes to elucidate observed data patterns. This approach typically disregards the persistent component of earnings shocks, allowing variance to fluctuate across time and individuals. Key contributions in this area include the studies by Barsky et al. (1997) and Meghir and Pistaferri (2004).

This paper presents a RIP model that intersects both aforementioned categories by examining both the conditional mean and variance of the income process, akin to the analyses in Browning et al. (2010) and Hospido (2012). It further aligns with the burgeoning Bayesian approach to income dynamics, a field recently enriched by the contributions of Geweke and Keane (2000) and Norets and Schulhofer-Wohl (2010). The Bayesian framework's potential for capturing nuanced heterogeneity and its favorable small-sample properties, as highlighted by Nakata and Tonetti (2015), suggest a promising avenue for future research.

The role of aging in shaping idiosyncratic earnings shocks remains a relatively underexplored dimension. Pioneering efforts in this domain include the work of Hause (1980), who investigated the impact of "on-the-job training" on earnings profiles within a Swedish cohort, implicitly addressing age-related variations through a time-varying AR process. Meghir and Pistaferri (2004) introduced an ARCH(1) specification to model the conditional variance of labor earnings, though age did not emerge as a significant factor in their analysis.

The study by Karahan and Ozkan (2013) represents a comprehensive attempt to explicitly model the age profile of earnings shocks. By employing GMM methods, they discerned an age-related pattern in the persistence of earnings shocks, albeit not in transitory shocks. Their methodology involved estimating persistence parameters and variances for various age bins and subsequently simplifying the model with an age polynomial.

Distinct from Karahan and Ozkan (2013), this paper adopts a Bayesian approach, offering a more granular level of heterogeneity and the flexibility to extend to HIP settings. Unlike the multiple estimates necessitated by Karahan and Ozkan (2013)'s frequentist approach, this paper proposes a model characterized by two distinct regimes connected by a smooth transition function. The findings diverge as well; while Karahan and Ozkan (2013) observed significant variance in persistence and variances, this study suggests that the primary fluctuations occur in the innovations.

Crucially, this paper extends beyond age as the sole explanatory variable, accommodating additional factors to elucidate observed data patterns, thereby engaging with various economic theories.

In the following section, I start with a description of the statistical model, and Section III explains the estimation strategy. Section IV describes the data set and the selection criteria for the sample used. Section V presents the results, and Section VI concludes.

2. LSTAR Model for Income Dynamics

This study employs the Logistic Smooth Transition Autoregressive (LSTAR) model of order 1 to analyze the nonlinear evolution of labor income throughout an individual's career. I express the logarithm of labor earnings, $y^i_{h(t),t}$, for an individual i at labor age $h(t)$ during year t. The timeline spans from the individual's initial appearance in the PSID at t_i to their final observation at T_i. Labor age $h(t)$ is adjusted to commence at 0 for calendar age 24, extending up to H, the maximum labor age equivalent to calendar age 64.

The model captures earnings as influenced by time effects, individual characteristics $x^i_{h(t),t}$, a transient shock $\omega^i_{h(t),t}$ reflecting measurement errors and short-term productivity variations, and a lasting shock $\varepsilon^i_{h(t),t}$:

$$y^i_{h(t),t} = x^{i\prime}_{h(t),t}\beta + \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \tag{1}$$

with $\beta \in \Re^k$. The persistent shock, conventionally modeled as an AR(1) process, is re-envisioned here to accommodate a nonlinear pattern across the life cycle with two distinct regimes, transitioning smoothly via an LSTAR(1) framework for greater economic authenticity:

$$\varepsilon^i_{h(t+1),t+1} = (1 - G(\gamma, c, \tau^i_t))\rho_1\varepsilon^i_{h(t),t} + G(\gamma, c, \tau^i_t)\rho_2\varepsilon^i_{h(t),t} + \eta^i_{t+1} \tag{2}$$

$G(\gamma, c, \tau)$, the logistic cumulative distribution function, facilitates a smooth regime transition, contingent on the individual-specific variable τ^i_t and a threshold c. The smoothness parameter γ dictates the transition's graduality, with lower values leading to more gradual changes and higher values approximating the step function of a TAR model.

The model structure is presented as a state-space framework:

$$\begin{cases} y^i_{h(t),t} = x^{i\prime}_{h(t),t}\beta + \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \\ \varepsilon^i_{h(t+1),t+1} = \varepsilon^i(\gamma, c)_{h(t),t}\widetilde{\rho} + \eta^i_{t+1} \end{cases} \tag{3}$$

Here, $\varepsilon^i(\gamma, c)_{h(t),t}$ combines the persistent shock and its regime-dependent counterpart, with $\widetilde{\rho} = (\rho_1, \rho_2 - \rho_1)'$ capturing the regime-specific persistence.

The LSTAR(1) model, tailored to capture the nuanced income dynamics within the PSID, assumes a finite Gaussian mixture for the shock distribution ξ^i_t. This approach is grounded in the premise that any continuous distribution can be closely approximated by such a mixture, offering a versatile framework for depicting income variations over time.[1]

Expressly, the shock distribution is modeled as:

$$\xi_t^i \sim \sum_{j=1}^{m} p_{g_j} N\left(0, g_j^{-1}\right) \tag{4}$$

Let s_t^i, ranging from 1 to m, denotes an indicator for the specific Gaussian distribution from which individual i's shock at time t is drawn. This modeling choice allows for a nuanced representation of shock variances as a convex combination of variances from distinct regimes:

$$Var\left(\eta_t^i / s_t^i\right) = \left(\kappa_i g_{s_t^i}\right)^{-1} \left[\left(1 - G\left(\gamma, c, \tau_t^i\right)\right) + \phi G\left(\gamma, c, \tau_t^i\right)\right] \tag{5}$$

with $k_t^i(\gamma, c, \phi)$ encapsulating the transition function's influence.

To address the early-life determinants of lifetime earnings disparities—a significant portion of which is established before labor market entry—the model incorporates a scaling factor λ for the initial age-period shocks:

$$\eta_{1,t}^i = \lambda^{-0.5}\left(\kappa_i^{-0.5} \xi_t^i \sqrt{1 - G(\gamma, c, \tau_t^i) + \phi G(\gamma, c, \tau_t^i)}\right) \tag{6}$$

This feature is informed by empirical evidence suggesting the pre-market factors' substantial role in shaping income trajectories (refer to Keane and Wolpin (1997), Storesletten et al. (2004), and Huggett et al. (2011) for further insights).

In sum, the LSTAR(1) model presents a sophisticated and economically plausible framework for exploring the complex, nonlinear patterns of income dynamics over the lifespan. By integrating a smooth transition mechanism and accounting for pre-labor market influences, the model offers a nuanced alternative to conventional AR(1) and TAR models, providing deep insights into the dynamics of income.

[1]For a comprehensive discussion on finite mixture models and their applications, see Everitt and Hand (1981), Titterington et al. (1985), and McLachlan and Peel (2004).

2.1. Restricted Income Profile Model

In the empirical macro-literature on income, the Restricted Income Profile (RIP) model stands out for its specific approach to modeling labor income dynamics, alongside its counterpart, the Heterogeneous Income Profile (HIP) model. The RIP model posits that the growth rates of income, influenced by an additional year of labor market experience or alternatively age, are uniform across individuals. This contrasts with the HIP model, which allows for individual-specific growth rates, introducing a random-effect structure in place of the fixed-effect framework of the RIP model.

Despite the possibility of extending individual heterogeneity to the cubic polynomial of labor market experience, evidence from Baker (1997) and Guvenen (2009) suggests that such extensions do not significantly enhance model fit. The prevailing view is to maintain a simplified HIP model structure, assuming that income effects attributable to labor market experience powers are universally applicable.

One notable distinction is that a HIP model exhibits reduced persistence in the AR(1) process compared to a RIP model, a difference that bears significant implications for calibrating macro models with incomplete markets.

The formulation of a RIP model is succinctly captured as follows:

$$\begin{aligned} y^i_{h(t),t} &= x^{i\prime}_{h(t),t}\beta + \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \\ y^i_{h(t),t} &= \beta_0 + \beta_1 h(t) + \widetilde{x}^{i\prime}_{h(t),t}\beta_2 + \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \end{aligned}$$

where $x^i_{h(t),t} = (1, h(t), \widetilde{x}^{i\prime}_{h(t),t})'$, and $\beta = (\beta_0, \beta_1, \beta_2)'$, with $\beta_2 \in \Re^{k-2}$.

In contrast, the HIP model is characterized by:

$$\begin{aligned} y^i_{h(t),t} &= \widetilde{\beta}^i_0 + \widetilde{\beta}^i_1 h(t) + \widetilde{x}^{i\prime}_{h(t),t}\beta_2 + \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \\ y^i_{h(t),t} &= x^{i\prime}_{h(t),t}\beta^i + \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \end{aligned}$$

where $\beta^i = (\widetilde{\beta}^i_0, \widetilde{\beta}^i_1, \beta_2)'$, and $(\alpha^i_0, \alpha^i_1)' \sim G(\cdot)$, with $E(\alpha^i_0) = 0$, $E(\alpha^i_1) = 0$, and $Var[(\alpha^i_0, \alpha^i_1)'] = H^{-1}_{\alpha}$.

Although economic theory may lean towards the HIP model due to the realistic portrayal of individual income growth rates driven by differences in ability, the empirical evidence remains mixed, as highlighted by Abowd and Card (1989) and Hryshko (2012). This paper, however, centers on the RIP model, particularly focusing on age-dependent shocks and delineating its distinctions from the HIP framework.

3. Estimation Methodology

This paper explores three distinct models to analyze income dynamics: a conventional Restricted Income Profile (RIP) model, a RIP model incorporating constant persistence with heterogeneity in the shock components, and a more complex RIP model that accounts for heterogeneity in both persistence and the shocks.

For each model, the analysis includes a cubic polynomial of work experience to model the characteristic hump-shaped trajectory of mean earnings throughout the life cycle. This is complemented by variables for time effects, educational attainment, marital status, racial background, and a binary indicator identifying individuals as partially-retired, following the approach outlined in Casanova (2013).[2]

In the traditional RIP model, after adjusting for observable characteristics, I pinpoint the necessary moment conditions within the autocovariance matrix and employ the Generalized Method of Moments (GMM) for estimation. For the subsequent models, I adopt a Bayesian framework, estimating their posterior distributions through Markov Chain Monte Carlo (MCMC) methods.

3.1. Prior Selection

The selection of priors is guided by the availability of prior information, opting for conjugate or non-informative priors accordingly. The chosen priors are standard within the field, with a comprehensive breakdown and hyperparameter details provided in Appendix B. The priors employed are as follows:

[2] Individuals over the age of 55 are classified as partially-retired if their hourly wage falls below 70% of the average wage observed for the 50-55 age group, with this reduced wage persisting until age 64.

1. The error and transitory productivity components, denoted by $\omega^i_{h(t),t}$, are assumed normal with $\omega^i_{h(t),t}/\sigma^2_t \sim N(0, \sigma^2_t)$, and the variance scale parameter $\underline{s}^2_\omega(\sigma^2_t)^{-1} \sim \chi^2(\nu_\omega)$.

2. Uninformative priors are set for the threshold variable c and the proportionality factor ϕ, with $p(c) \propto 1$ and $p(\phi) \propto \frac{1}{\phi}$.

3. The smoothing parameter γ follows a truncated Cauchy distribution, ensuring positivity: $p(\gamma) \propto \begin{cases} (1+\gamma^2)^{-1}, & \gamma > 0 \\ 0, & \text{otherwise} \end{cases}$.

4. Conjugate priors are selected for the state variables s^i_t and the probability vector $\underline{\mathbf{p}}$: $s^i_t|\underline{\mathbf{p}} \sim$ Multinomial($\underline{\mathbf{p}}$), and $\underline{\mathbf{p}} \sim$ Dirichlet($\boldsymbol{\alpha}$).

5. The persistence parameter $\widetilde{\rho}$ is normally distributed with mean $\underline{\mu}_{\widetilde{\rho}}$ and precision $h_\rho I$: $\widetilde{\rho} \sim N(\underline{\mu}_{\widetilde{\rho}}, h^{-1}_\rho I)$.

6. The regression coefficients β follow a normal distribution with hyperparameters μ and H: $\beta|(\mu, H) \sim N(\mu, H^{-1})$, where $\mu \sim N(\underline{\mu}_\mu, \underline{H}^{-1}_\mu)$ and $H \sim$ Wishart($\underline{\nu}_H, \underline{S}_H$).

7. The variance parameters for individual-specific random effects κ_i and the common shock λ are chi-squared distributed: $s^2_\kappa \kappa_i \sim \chi^2(\underline{\nu}_\kappa)$ and $s^2_\lambda \lambda \sim \chi^2(\nu_\lambda)$.

3.2. Posterior Distribution

The full posterior distribution for the model under study is defined as:

$$p(\Theta|y) = p\left(y^*, \{\varepsilon^i_{h(t),t}\}^{I,T_i}_{i=1,t=t_i}, \beta, \{\sigma^2_t\}^T_{t=1}, \rho_1, \rho_2, \gamma, c, \{\kappa_i\}^I_{i=1}, \lambda, \mu, H, \mathbf{p}, \{g_j\}^m_{j=1}, \{s^i_t\}^{I,T_i}_{i=1,t=t_i}|y\right) \tag{7}$$

where $y_i = (y^i_{h(t_i)t_i}, ..., y^i_{h(T_i)T_i})'$ and $y = (y'_1, ..., y'_I)'$. The imputed values $y^{*,i}_{h(t),t}$ are used for missing or top-coded observations, i.e.

$$y^i_{h(t),t} = \begin{cases} y^i_{h(t),t}, & \text{when the observation is present in the sample;} \\ y^{*,i}_{h(t),t}, & \text{when the observation is not present in the sample;} \end{cases}$$

The object 7 has all the necessary information to infer the income dynamics in the sample.

3.3. Markov Chain Monte Carlo Techniques

Markov Chain Monte Carlo (MCMC) methods are a class of simulation techniques that generate sequences of random samples, where each sample depends on the preceding one, forming a Markov Chain. These sequences are designed to converge to a stable distribution, which represents the target of the investigation (refer to Robert and Casella (2013) for a comprehensive overview). Among the plethora of MCMC techniques, the Gibbs Sampler and the Metropolis-Hastings algorithm stand out for their effectiveness in examining posterior distributions.

3.3.1. The Gibbs Sampler

The Gibbs Sampler, as introduced by Geman and Geman (1984), employs a strategy of partitioning the target distribution 7 and sequentially sampling from each conditional distribution of a partition, given the others. To elucidate, consider the posterior distribution $p(\theta|y)$, where $\theta \in \Re^k$ represents the parameter vector and $y \in \Re^n$ symbolizes the observed data. The parameter vector θ is divided into segments θ_0 and θ_1. Starting from an initial state $\theta^{(0)} = (\theta_0^{(0)}, \theta_1^{(0)})'$, the Gibbs Sampler iteratively updates each segment by drawing from its conditional distribution given the other segment: for each iteration $m \in \{1, \ldots, B\}$, $\theta_0^{(m)} \sim p(\theta_0|\theta_1^{(m-1)}, y)$ and $\theta_1^{(m)} \sim p(\theta_1|\theta_0^{(m)}, y)$. This process generates a Markov Chain that, given sufficient iterations, converges to the desired posterior distribution $p(\theta|y)$.

In the context of this model, it is practical to organize the sampling process into 15 distinct blocks. Twelve of these blocks lend themselves readily to Gibbs Sampling due to their straightforward conditional distributions. The remaining three, however, involve more complex distributions that necessitate integrating a Metropolis-Hastings step within the Gibbs Sampling framework. The comprehensive derivations of the posterior distributions for each partition are detailed in Appendix A.

The Gibbs Sampling procedure unfolds through the following 12 steps:

1. Sample the imputed values y^*, given the model parameters β, the variances $\{\sigma_t^2\}_{t=1}^T$, the covariates $\{x_{h(t),t}^i\}_{i,t}$, the state indicators $\{s_t^i\}_{i=1,t=t_i}^{I,T_i}$, the state-specific variances $\{g_j\}_{j=1}^m$, the autocorrelation parameters ρ_1, ρ_2, the smoothing parameter γ, the threshold c, the scaling factor ϕ, and the observed data y.

2. Update the individual-specific shocks $\{\varepsilon_{h(t),t}^i\}_{i=1,t=t_i}^{I,T_i}$, conditional on the observed and imputed data, model parameters, and the rest of the latent variables.

3. Update the regression coefficients β, based on the observed data, imputed values, covariates, shocks, and prior information on β (mean μ and precision matrix H).

4. Update the variance terms $\{\sigma_t^2\}_{t=1}^T$, considering the observed and imputed data, covariates, and shocks.

5. Update the persistence parameters $\widetilde{\rho}$, given the shocks and other model parameters.

6. Update the individual-specific random effects $\{\kappa_i\}_{i=1}^I$, considering the shocks and other model components.

7. Update the common shock variance λ, based on the first-period shocks and the random effects.

8. Update the hyperparameter μ of the regression coefficients' prior distribution, given the current estimate of β.

9. Update the precision matrix H of the regression coefficients' prior distribution, based on the current estimate of β and its mean μ.

10. Update the probabilities $\mathbf{p}$ for the state indicators, given their current assignments.

11. Update the state indicators $\{s_t^i\}_{i=1,t=t_i}^{I,T_i}$, considering the model's shocks, random effects, and other parameters.

12. Update the state-specific variance parameters $\{g_j\}_{j=1}^m$, based on the state indicators, shocks, and other model components.

3.3.2. Metropolis-Hastings within the Gibbs Sampler

Within the Gibbs sampling framework, three posterior distributions—referred to as blocks—require sampling to complete the model's posterior distribution analysis. The Metropolis-Hastings algorithm, as introduced by Metropolis et al. (1953) and Hastings (1970), offers a robust approach for this purpose.

The Metropolis-Hastings algorithm, a cornerstone of MCMC methods, operates by generating candidate samples from a proposal distribution $q(\cdot)$ and accepting these samples based on their likelihood under the target distribution $p(\cdot)$. This selective acceptance ensures that the ensemble of accepted samples approximates the target distribution effectively.

To formalize, consider a target distribution $p(x)$ supported over $D \in \Re^k$, with x and proposed x^* both residing in D. During the m^{th} iteration, the algorithm evaluates the candidate x^*, drawn from $q(x^*/x)$, against the current state x. The acceptance probability for x^* is given by:

$$\alpha(x^*, x) = \min\left\{\frac{p(x^*)}{p(x)}\frac{q(x/x^*)}{q(x^*/x)}, 1\right\}. \tag{8}$$

This criterion ensures that samples moving towards higher probability regions are favored, promoting convergence to the target distribution. The Metropolis-Hastings algorithm's reversibility, guaranteed by this acceptance rule, is critical for the Markov chain's convergence to the desired distribution (detailed discussions can be found in Chib and Greenberg (1995) and Geweke (2005)).

For this model's implementation, the three challenging posterior distributions are approached using an Independent Metropolis-Hastings scheme, characterized by proposal densities $q(x^*/x)$ that are independent of the current state x. This simplifies the acceptance probability to:

$$\alpha(x^*, x) = \min\left\{\frac{p(x^*)}{p(x)}\frac{q(x)}{q(x^*)}, 1\right\}.$$

Sampling strategies for these distributions utilize truncated normal distributions for γ^* and ϕ^*, and a discrete distribution for c^*, ensuring a balanced exploration of the state space. The proposal parameters are fine-tuned to achieve an optimal acceptance rate of approximately 30%, enhancing the Markov chain's efficiency (as recommended by Müller (1991) and Canova (2007)).

4. Panel Study of Income Dynamics (PSID)

The Panel Study of Income Dynamics (PSID) represents a comprehensive panel dataset, encompassing over 9,000 families and 70,000 individuals tracked continuously since 1968. It stands as the world's most extensive and enduring household-based survey, providing an unparalleled resource for analyzing income dynamics within the United States.

The focus of my analysis is on male household heads from 1968 to 2019, subject to the following criteria:

1. Possession of at least two consecutive observations of earnings;
2. Age range between 25 and 64 years at the time of survey participation;
3. Exclusion from the Survey of Economic Opportunities (SEO) sample;
4. Positive hourly wages and labor income;[3]
5. Hourly wages ranging from \$2.63 to \$600 in 2007 dollars;[4]
6. Annual work hours between 456 and 4,992;[5]
7. Consistent and complete data on years of education;

[3] Labor income variables from the PSID include: V74, V514, V1196, V1897, V2498, V3051, V3463, V3863, V5031, V5627, V6174, V6767, V7413, V8066, V9376, V8690, V11023, V12372, V13624, V14671, V16145, V17534, V18878, V20178, V21484, V23323, (ER4140+ER4117+ER4119), (ER6980+ER6957+ER6959), (ER9231+ER9208+ER9210), and (ER12080+ER12065+ER12193).

[4] The CPI from the US Department of Labor, Bureau of Labor Statistics is used to adjust labor income series for inflation, available at ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt

[5] The upper limit of 4,992 hours is set to exclude likely erroneous reports. This equates to 16-hour workdays, six days a week, which is generally unsustainable.

A significant aspect of the PSID to consider is its shift to biannual data collection post-1997. To address this change, I have imputed missing income values, detailed in the Appendix.

Upon rectifying inconsistencies, the refined sample comprises 5,398 individuals, resulting in 61,163 individual-year observations (refer to Table 1). On average, individuals are observed over an 11-year span. Table 1 reveals that the average and median hourly wages are \$26.51 and \$31.66 respectively, with a standard deviation of \$22.15 and an interquartile range of \$16.37, adjusted to 2007 dollar values.

The earnings distribution displays the expected right-skew typical of income data. Examination of the median values in Figure 1's boxplots, highlighted in red, illustrates the conventional earnings peak observed in life-cycle income trajectories. The boxplot widths, indicative of the interquartile range, offer a robust measure of income variation over the life span, expanding into the mid-30s, stabilizing through the 40s, and narrowing beyond.

This variation in income dispersion may reflect life-cycle decisions in human capital accumulation (refer to Ben-Porath (1967)), occupational mobility and the associated risks and returns (Kambourov and Manovskii (2009), Cubas and Silos (2012)), intra-household resource allocation shifts (Greenwood et al. (2003), Knowles (2013)), or strategies like returning to parental homes as a buffer against adverse financial shocks (Kaplan (2012)).

Understanding the underlying factors in Figure 1c, whether they pertain to human capital investment, risk management, or family dynamics, is crucial for accurately modeling income dynamics across the life cycle.

5. Results

For the analysis utilizing the LSTAR(1) model, the Markov Chain Monte Carlo (MCMC) sampling process involves generating a total of 50,000 iterations. Observations indicate that the chain reaches convergence after approximately 5,000 iterations. The initial 5,000 iterations, identified as the burn-in phase, are subsequently excluded from the analysis. Trace plots from the remaining samples exhibit satisfactory mixing, suggesting effective exploration of the posterior distribution.

In contrast, the analysis for the Restricted Income Profile (RIP) model, which incorporates constant persistence alongside innovation heterogeneity, utilizes a smaller MCMC sample size of 20,000 iterations. Similar to the LSTAR(1) model, convergence appears to be achieved after the first 5,000 iterations, which are also designated as the burn-in period and discarded.

To ensure robustness in the convergence assessment, the sampling process is replicated multiple times from varied and widely spaced initial values, following the recommendations of Gelman and Rubin (1992). These repeated samplings yield consistently favorable results, reinforcing the reliability of the convergence findings. Additionally, the convergence of the MCMC chains is further validated through Geweke's convergence diagnostic (refer to Geweke et al. (1991)), which corroborates the stability and convergence of the sampling process across both models.

5.1. Constant persistence and heterogeneity in the innovations

5.1.1. 1-Regime Model with Normal Errors

GMM techniques, leveraging the method of moments, provide a foundational approach in traditional models, equating theoretical population moments with empirical sample moments across diverse data distributions. In anticipation, it's imperative to ascertain whether Bayesian estimation with a normal and rich innovation structure aligns with GMM-derived outcomes. To this end, I juxtapose our model's Bayesian estimations against those obtained from a conventional Restricted Income Profile (RIP) model utilizing GMM, a staple in empirical macro-literature and dynamic general equilibrium model calibrations.

The concurrence between Bayesian methods and GMM estimations is affirmed, even within models assuming Normally distributed errors. Table 2 delineates that idiosyncratic earnings shocks exhibit notable persistence in the life cycle, with a persistence parameter ρ at 0.97, a persistent shock variance σ_η^2 at 0.029, and a temporary shock variance σ_ω^2 at 0.095. This stationary AR(1) process, verging on a unit root, implies that a shock's effect diminishes by half over 17 periods, suggesting that a worker endures over 40% of their working life under the influence of an incompletely mitigated shock. Bayesian analysis reinforces this, indicating a 99% probability for ρ within the 0.96 to 0.98 range, aligning with prevalent literature estimates.

While this model presupposes a common income growth rate across agents, the initial earnings level varies, encapsulated by an intercept α with a variance σ_α^2 estimated at 0.061. The sum of σ_α^2 and σ_η^2 quantifies the idiosyncratic earnings shock variance in the initial age period sans the transitory variance component, equating to 0.09. This suggests a substantial 29% earnings increase from a one-standard-deviation shock relative to the mean.[6]

Delving into individual profiles via Table 3 reveals a majority with κ^{-1} below one, indicating predominantly low-risk individuals, alongside a subset of "risky" individuals. Additionally, Initial Variance Estimates exhibit discrepancies between the mean and mode at the labor market's inception, with the mean Variance at First Age at 0.063, closely mirroring GMM estimates, yet the mode Variance at First Age stands at 0.013, hinting at potential biases.

Key Insights: A significant majority encounters less income volatility than standard models suggest, indicating an overestimation of risk in conventional approaches. The variance in κ_i underscores the criticality of accommodating individual heterogeneity in income shock modeling.

The first comparison involves a 1-Regime LSTAR(1) model, akin to the previously discussed, albeit with a singular regime, implying constant persistence across the life cycle and a nuanced innovation heterogeneity. As Table 2 illustrates, negligible discrepancies exist between these models' estimates, with Bayesian settings offering reduced variances compared to GMM.[7] This underscores that the innovation structure's assumptions minimally impact the standard models' conclusions regarding shock variance and persistence.

Table 2 highlights the posterior distribution of the persistence parameter ρ, show-

[6] Labor earnings in efficiency units are modeled as $e^x w\theta$, where w denotes the wage rate, θ the hours worked, and $x \sim N\left(\mu, \sigma^2\right)$ the earnings shocks. Thus, the efficiency units e^x manifest as a log-normal variable with mean $e^{\mu+\sigma^2/2}$. A one-standard-deviation shock from mean μ elevates labor earnings by $e^{\sigma-\sigma^2/2}$.

[7] This observation resonates with findings in Nakata and Tonetti (2015).

casing a mode at 0.97 and a standard deviation of 0.003. Figure 2a confirms a 99% likelihood for ρ within the 0.96 to 0.98 range, endorsing the GMM's ρ estimate of 0.976 within this model framework.

The parameters g^{-1}, κ_i^{-1}, and λ^{-1} constitute the innovations' variance components. The posterior distributions of λ^{-1} and g^{-1} are depicted in Figures 3a and 3b, respectively, with κ_i^{-1}'s boxplot presented in Figure 3c. Notably, over 50% of individuals exhibit κ_i^{-1} values under 1, as delineated in Table 5. However, the density plot in Figure 3c reveals individuals with substantially higher conditional variances, unaccounted for in traditional RIP models, potentially leading to overestimated variance in conventional RIP estimations. For instance, the mode of initial shock variance, derived from the mode of g^{-1}, κ_i^{-1}, and λ^{-1}, approximates 0.06, aligning with conventional model estimates. Yet, focusing on the mean distributions elevates the initial age-period variance to 0.26, deviating from previous estimates.

This discrepancy bears significant economic interpretation: a one-standard-deviation shock could imply a 46% earnings increase relative to mean earnings, considering the mean variance components, contrasting with a mere 23% increase based on the mode. This distinction underscores the necessity of contextualizing findings, especially given the overwhelming majority of individuals with less risky income shock processes than standard models typically suggest. Further examination (see Figure 4 and Table 6) reveals no substantial correlation between ”risky” classification and individuals' race or marital status, with a notable prevalence among high school dropouts, postgraduates, and those in entrepreneurial or managerial roles, highlighting occupational choice's influence on income risk.

The similarity in initial shock dispersion across both models, coupled with the conventional RIP's overlook of κ_i heterogeneity, advises against standard specifications' sole reliance.

5.1.2. 1-Regime Model with Mixture of Normal Errors

The exploration into innovations reveals overlooked complexities, prompting a departure from normality assumptions towards a more adaptable mixture of Normal distributions. Preliminary Q-Q plot analyses suggest thinner tails than a normal distribution, indicating fewer extreme outliers and hinting at the inappropriateness of normality assumptions (see Figure 1a).

Employing a two-normal-distribution mixture refines the error distribution approximation, introducing higher kurtosis indicative of heavier tails and a pronounced peak, thus heightening the probability of extreme values. This new distributional framework elevates shock persistence beyond prior observations, suggesting that shocks have more enduring impacts (see Figure 2). A staggering 79% likelihood exists for workers to encounter low-variance shocks, interspersed with high-variance shocks approximately every five years, emphasizing the heavy-tailed, sharp-peaked distribution's propensity for more frequent extreme shocks than a normal distribution would predict (see Table 4).

This flexible distributional approach significantly reduces the classification of workers as "risky," with over 75% showcasing $\kappa_i^{-1} < 0.8$, denoting diminished risk levels. Yet, a minority still navigates considerable income volatility, as illustrated in Figure 6.

5.2. Heterogeneity in the persistence and the innovations

5.2.1. 2-Regimes Model: Age as Threshold Variable

Moving beyond the constant persistence assumption of standard models, this subsection contrasts the conventional Restricted Income Profile (RIP) with a Logistic Smooth Transition Autoregressive (LSTAR)(1) model. This LSTAR(1) framework delineates heterogeneity in both persistence and innovations, interconnected by a smooth transition function, with the variable age serving as a pivotal threshold. This setup allows the idiosyncratic shocks' nature to evolve across the life cycle, influenced by the proximity of age to the threshold.

The posterior distribution of the threshold variable c, illustrated in Figure 7a, and summarized in Table 9, indicates a significant probability of the threshold being below 31, pinpointing 29 as the mode. This underscores a clear life-cycle pattern in earnings shocks, with a substantial likelihood of transitioning to a new shock regime by age 31, nearly guaranteeing a transition by age 35. Interestingly, the transition age diverges based on education, occurring around age 45 for college graduates and age 55 for high school graduates, suggesting that education level significantly modifies the timing of shock regime shifts.

The transition function's smoothness is governed by parameter γ, with a posterior mode of 0.52 and a standard deviation of 0.487, as detailed in Table 9. An examination of Figure 7b reveals a 50% posterior probability for γ lying between 0.44 and 0.89, emphasizing the role of parameters γ and c in shaping the persistence parameters ρ and the variance of innovations.

A notable aspect is the evolution of the combined persistence parameter ρ across different ages, as outlined in Table 8. At 25, the persistence mode stands at 0.959, ascending to 0.97 by 45 and stabilizing thereafter. The variance in ρ is more pronounced at the onset of the life cycle, suggesting that age alone may not fully capture the dynamics at play, as indicated by the broad dispersion in Figure 10a and the significant probability mass in the $[0.955, 0.965] \times [0.965, 0.975]$ region of Figure 8a for joint distributions of ρ_1 and ρ_2.

The model's introduction of nonlinearities leads to a nuanced understanding of innovations' variance through parameters ϕ and $k_t^i(\gamma, c, \phi)$. Figure 11a and Table 10, alongside Figure 12b, elucidate how innovations' variance diminishes as individuals age, with a 50% posterior probability for ϕ ranging between 0.71 and 0.80. This translates to a 1.3 times higher stationary variance for younger workers compared to older counterparts, further diversified by education, with high school graduates witnessing a variance decline post-50, and college graduates experiencing a variance surge post-30.

The variance's individual-specific component κ_i^{-1} mirrors findings from the previous subsection, with a posterior mean of 2.37 and a substantial fraction of the sample exhibiting κ_i^{-1} values below 1.75, as seen in Figure 13a. The distribution of other variance components, λ and g, is depicted in Figures 12c and 12a respectively.

This model's initial variance mode at the first life-cycle stage is 0.06, implying a 23% earnings increase from a one-standard-deviation shock relative to the mean. However, considering the mean of variance components elevates the initial variance to 0.28, leading to a 47% earnings rise from a similar shock. This highlights the criticality of acknowledging individual-specific heterogeneity, which standard models often overlook, potentially leading to skewed interpretations of income dynamics.

5.2.2. 2-Regimes Model: Job Tenure as Threshold Variable

A closer examination of Figure 5 reveals that age alone does not fully encapsulate the dynamics of income shocks, particularly when observing the disparate effects on college and high school graduates. This discrepancy prompted an exploration of job tenure as a potential variable to provide a clearer understanding of the underlying forces.

The application of job tenure as a threshold variable in the model yields insightful findings. The distribution of changes in shock regimes, predominantly occurring within the first five years of employment, suggests a significant adjustment phase for workers, with the median transition observed after just one year on the job, as depicted in Figure 7b. Notably, this transition appears consistent across educational backgrounds, indicating a universal aspect of job tenure on earnings shocks.

Persistence of Shocks Across Job Tenure An intriguing pattern emerges when analyzing shocks' persistence relative to job tenure. Initially, shocks exhibit lower persistence, particularly among high school graduates, with a marked increase in persistence observed as tenure progresses, as detailed in Table 8 and illustrated in Figures 8b and 9b. This trend underscores a growing stability in earnings as workers gain tenure, with Figure 10b highlighting the increasing persistence of shocks with tenure.

Variance Components and Job Tenure Further analysis of variance components, presented in Tables 9 and 10, reveals a consistent pattern across educational groups, even when controlling for job tenure. However, a subset of individuals still exhibits higher risk profiles, as shown in Figure 13b. Notably, college graduates face a lower variance at age 25 compared to high school graduates, suggesting smoother transitions into the labor market for the former group. The variance of shocks in the initial tenure stages significantly exceeds those in later stages, with Figure 11b demonstrating a substantial decrease in shock variance after the third year of employment, emphasizing the stabilizing effect of job tenure on earnings dynamics, devoid of educational background distinctions.

Implications of Job Tenure on Earnings Shocks The analysis unequivocally establishes job tenure as a critical determinant of earnings shock dynamics, effectively nullifying the educational disparity in shock experiences. The transition from high volatility in early tenure to markedly reduced volatility in later stages, as shown in Figure 11b, highlights a significant shift towards increased job stability and earnings predictability with accrued tenure. This evolution reflects the profound impact of job tenure on the nature and frequency of earnings shocks, suggesting its pivotal role in shaping income dynamics over the life cycle.

6. Conclusion

In this paper, I have unveiled life-cycle patterns in idiosyncratic earnings shocks, pioneering the application of a Smoothed Transition Autoregressive (LSTAR) model to the persistent component of residual earnings. This innovative approach transcends traditional models by adeptly disentangling the economic forces at play across the life cycle, offering a simplified yet potent framework to capture the nuanced complexity inherent in income processes. Furthermore, the LSTAR model's adaptability enhances our macro models' calibration in two significant ways: by helping us understand the underlying economic mechanisms shaping income shock dynamics and by giving us an empirical process that can easily be incorporated into a calibration by standard approximation techniques.

Transitioning from the conventional GMM framework, this study embraces a Bayesian methodology, which is particularly adept at navigating the nonlinearities and heterogeneity of income dynamics. Bayesian methods are uniquely suited to the intricacies of the PSID dataset, with its biannual periodicity post-1997 and the presence of top-coded observations necessitating imputation. Bayesian methods not only facilitate up-to-date estimation of income shock processes but also simplify the estimation of income processes where the persistent component is not directly observable.

My findings reveal that while most shocks experienced by workers are of low variance and high persistence, episodes of high-variance shocks are not uncommon. I have shown a link between these shocks and age, with job tenure emerging as a pivotal factor in income dynamics. Still, many forces are at play, as evidenced by the differing income processes between high school and college graduates, indicating the need for more comprehensive modeling and the introduction of new variables.

A comparative analysis of a Restricted Income Profile (RIP) model with constant persistence and heterogeneity against a standard RIP model indicates that many individuals face less risky income shock processes than previously assumed. The standard model overlooks this heterogeneity, leading to an overestimation of variance. Our Bayesian specification addresses this data feature, recommending its consideration in macroeconomic model calibration.

Future research directions include extending this model to a Heterogeneous Income Profile (HIP) setting and incorporating additional threshold variables such as income level, economic recession indicators, occupational mobility, and sectoral employment. Model selection criteria can then be applied to identify the most accurate model for simulating earning paths across the life cycle. Comparing these simulations with standard model predictions and actual earnings data will illuminate the efficacy and economic implications of different models.

In conclusion, this paper contributes significantly to our understanding of income shocks by challenging conventional income dynamics models and highlighting the critical role of variables such as age and job tenure. The Bayesian estimation of LSTAR models not only facilitates overcoming the challenges posed by nonlinear income processes but also opens new avenues for enhancing macroeconomic analysis and policy formulation.

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Appendix A Posterior Distributions.

In this appendix, I derive the posterior distributions of the parameters of interest in each of the Gibbs sampler steps for a Heterogeneous Income Profile Process (HIP) and a Restricted Income Profile Process (RIP).

A1. Heterogeneous Income Profile Process

1. $\underline{\left(y^*/\left\{x^i_{h(t),t}\right\}_{i,t},\left\{\beta^i\right\}_{i=1}^N,\left\{\sigma_t^2\right\}_{t=1}^T,\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i},\left\{g_j\right\}_{j=1}^m,\rho_1,\rho_2,\gamma,c,\phi,y\right)'}$

   Our starting point is:

$$\begin{cases} y^i_{h(t),t}=x^{i\prime}_{h(t),t}\beta^i+\varepsilon^i_{h(t),t}+\omega^i_{h(t),t} \\ \varepsilon^i_{h(t),t}=\underbrace{\left[\left(1-G\left(\gamma,c,\tau_t^i\right)\right)\rho_1+G\left(\gamma,c,\tau_t^i\right)\rho_2\right]}_{m_t^i(\gamma,c,\rho_1,\rho_2)}\varepsilon^i_{h(t)-1,t-1}+\eta_t^i \end{cases}$$

   Then,

   $\varepsilon^i_{h(t),t}=y^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta^i-\omega^i_{h(t),t}$, and

   $y^i_{h(t),t}=x^{i\prime}_{h(t),t}\beta^i+m_t^i\left(\gamma,c,\rho_1,\rho_2\right)\varepsilon^i_{h(t)-1,t-1}+\eta_t^i+\omega^i_{h(t),t}$

   Combining both equations, we have:

   $y^i_{h(t),t}=x^{i\prime}_{h(t),t}\beta^i+m_t^i\left(\gamma,c,\rho_1,\rho_2\right)\left(y^i_{h(t)-1,t-1}-x^{i\prime}_{h(t)-1,t-1}\beta^i-\omega^i_{h-1,t-1}\right)+\eta_t^i+\omega^i_{h(t),t}$

   $y^i_{h(t),t}=\left(x^i_{h(t),t}-m_t^i\left(\gamma,c,\rho_1,\rho_2\right)x^i_{h(t)-1,t-1}\right)'\beta^i+m_t^i\left(\gamma,c,\rho_1,\rho_2\right)y^i_{h(t)-1,t-1}+\omega^{i\prime}_{h(t),t}-m_t^i\left(\gamma,c,\rho_1,\rho_2\right)\omega^i_{h-1,t-1}+\eta_t^i$

   Using the above, we can find the posterior distribution:

   $p\left(y^{*,i}_{h(t),t}/\left\{x^i_{h(t),t}\right\}_{i,t},\left\{\beta^i\right\}_{i=1}^N,\left\{\sigma_t^2\right\}_{t=1}^T,\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i},\left\{g_j\right\}_{j=1}^m,\rho_1,\rho_2,\gamma,c,\phi,y\right)\propto$

   $\propto p\left(y^i_{h(t)+1,t+1}/y^{*,i}_{h(t),t},\cdots\right)p\left(y^{*,i}_{h(t),t}/y^i_{h(t)-1,t-1},\cdots\right)$

$$\propto \exp\left\{-\frac{1}{2}\left\{\begin{array}{c}\left(\underbrace{\sigma_{t+1}^2+m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)^2\sigma_t^2+Var\left(\eta_{t+1}^i\right)}_{R_{t+1}}\right)^{-1}\times\\ \times\left[\begin{array}{c}y_{h(t)+1,t+1}^i-\left(x_{h+1,t+1}^i-m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)x_{h(t),t}^i\right)'\beta^i-\\ -m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)y_{h(t),t}^{*,i}\end{array}\right]^2+\\ +R_t^{-1}\left[\begin{array}{c}y_{h(t),t}^{*,i}-\left(x_{h(t),t}^i-m_t^i\left(\gamma,c,\rho_1,\rho_2\right)x_{h(t)-1,t-1}^i\right)'\beta^i-\\ -m_t^i\left(\gamma,c,\rho_1,\rho_2\right)y_{h(t)-1,t-1}^i\end{array}\right]^2\end{array}\right\}\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left\{\begin{array}{c}\left(y_{h(t),t}^{*,i}\right)^2\left[\left(m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)\right)^2R_{t+1}^{-1}+R_t^{-1}\right]-\\ -2y_{h(t),t}^{*,i}\left[\begin{array}{c}R_{t+1}^{-1}m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)\times\\ \times\left(y_{h(t)+1,t+1}^i-\left(x_{h+1,t+1}^i-m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)x_{h(t),t}^i\right)'\beta^i\right)+\\ +R_t^{-1}\left(\begin{array}{c}\left(x_{h(t),t}^i-m_t^i\left(\gamma,c,\rho_1,\rho_2\right)x_{h(t)-1,t-1}^i\right)'\beta^i+\\ +m_t^i\left(\gamma,c,\rho_1,\rho_2\right)y_{h(t)-1,t-1}^i\end{array}\right)\end{array}\right]\end{array}\right\}\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\overline{h}_y\left(y_{h(t),t}^{*,i}-\overline{\mu}_y\right)^2\right\}$$

where

$$\overline{h}_y=\left(m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)\right)^2R_{t+1}^{-1}+R_t^{-1}$$

and

$$\overline{\mu}_y=\overline{h}_y^{-1}\left\{\begin{array}{c}R_{t+1}^{-1}m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)\times\\ \times\left(y_{h(t)+1,t+1}^i-\left(x_{h+1,t+1}^i-m_{t+1}^i\left(\gamma,c,\rho_1,\rho_2\right)x_{h(t),t}^i\right)'\beta^i\right)+\\ +R_t^{-1}\left[\left(x_{h(t),t}^i-m_t^i\left(\gamma,c,\rho_1,\rho_2\right)x_{h(t)-1,t-1}^i\right)'\beta^i-m_t^i\left(\gamma,c,\rho_1,\rho_2\right)y_{h(t)-1,t-1}^i\right]\end{array}\right\}$$

Therefore,

$$y_{h(t),t}^{*,i}/\cdots\sim N\left(\overline{\mu}_y,\overline{h}_y^{-1}\right).$$

For the time-period when the PSID becomes bi-annual, I impute the values for the missing years with $y_{h(t),t}^{*,i}/\cdots\sim N\left(\overline{\mu}_y,\overline{h}_y^{-1}\right)I\left(y_{h(t),t}^{*,i}\right)_{[\underline{y},\overline{y}])}$, where $\underline{y}$ and $\overline{y}$

are the lower- and upper-bounds defined in the text.

2. $\left(\left\{\varepsilon^i_{h(t),t}\right\}_{i=1,t=t_i}^{I,T_i}/\ldots\right)$

Following Durbin and Koopman (2002), we have for each individual i, conditional on $\{s^i_t\}^{T_i}_{t_i}$, a conditionally linear Gaussian state-space model given by:

$$\begin{cases} \widetilde{y}^i_{h(t),t} = y^i_{h(t),t} - x^{i\prime}_{h(t),t}\beta^i = \varepsilon^i_{h(t),t} + \omega^i_{h(t),t} \\ \varepsilon^i_{h(t),t+1} = m^i_{t+1}(\gamma, c, \rho_1, \rho_2)\,\varepsilon^i_{h(t),t} + \eta^i_{t+1} \end{cases} \quad (A.1)$$

where $\omega^i_{h(t),t} \sim N\left(0, \sigma^2_t\right)$, $\eta^i_t/s^i_t \sim N\left(0, g^{-1}_{s^i_t}\right)$ and $\varepsilon_{t_i-1} \equiv 0$, for $t_i \leq t \leq T_i$.

Let $w^i = \left(\omega^i_{h(t_i),t_i}, \eta^i_{t_i}, \ldots, \omega^i_{h(T_i),T_i}, \eta^i_{T_i}\right)'$, $\widetilde{y}^i = \left(\widetilde{y}^i_{h(t_i),t_i}, \ldots, \widetilde{y}^i_{h(T_i),T_i}\right)'$ and $\varepsilon^i = \left(\varepsilon^i_{h(t_i),t_i}, \ldots, \varepsilon^i_{h(T_i),T_i}\right)'$ where $w^i \sim N(0, \Omega)$, with $\Omega = diag\left(\sigma^2_{t_i}, g^{-1}_{s^i_{t_i}}, \ldots, \sigma^2_{T_i}, g^{-1}_{s^i_{T_i}}\right)$.

Step 1: Take a random draw from $w^+ \sim N(0, \Omega)$ and use it to generate $\widetilde{y}^{i+}$ and ε^{i+} from the recursion $(A.1)$.

Step 2: Apply the Kalman filter (Kalman, 1960) and the disturbance smoother to $\widetilde{y}^{i+}$ and $\widetilde{y}^i$:

Let

$$\begin{aligned} \widehat{\varepsilon}^i_{h(t),t/t-1} &= E\left(\varepsilon^i_{h(t),t}/\widetilde{y}^i_{h(t)-1,t-1}, s^i_t\right) \\ e^i_t &= \widetilde{y}^i_{h(t),t} - \widehat{\varepsilon}^i_{h(t),t/t} \\ R_{t/t-1} &= Var\left(\varepsilon^i_{h(t),t}/\widetilde{y}^i_{h(t)-1,t-1}, s^i_t\right) \\ &= \left(m^i_t(\gamma, c, \rho_1, \rho_2)\right)^2 R_{t-1/t-1} + g^{-1}_{s^i_t} \\ P_{t/t-1} &= Var\left(\widetilde{y}^i_{h(t),t}/\widetilde{y}^i_{h(t)-1,t-1}, s^i_t\right) \\ &= R_{t/t-1} + \sigma^2_t \end{aligned}$$

As,

$$\left[\begin{pmatrix} \widetilde{y}^i_{h(t),t} \\ \varepsilon^i_{h(t),t} \end{pmatrix} /\widetilde{y}^i_{h(t)-1,t-1}, s^i_t\right] \sim N\left(\begin{pmatrix} \widehat{\varepsilon}^i_{h(t),t/t-1} \\ \widehat{\varepsilon}^i_{h(t),t/t-1} \end{pmatrix}, \begin{pmatrix} P_{t/t-1} & R_{t/t-1} \\ R'_{t/t-1} & R_{t/t-1} \end{pmatrix}\right)$$

Then,

$$\begin{aligned} \widehat{\varepsilon}^{i}_{h(t),t/t} &= \widehat{\varepsilon}^{i}_{h(t),t/t-1} + \underbrace{R_{t/t-1}P^{-1}_{t/t-1}}_{K_t} e^{i}_{t} \\ R_{t/t} &= R_{t/t-1} - R_{t/t-1}P^{-1}_{t/t-1}R_{t/t-1} \\ &= \left(I - K_t\right) R_{t/t-1} \end{aligned}$$

The disturbance smoother is given by:

$$\widehat{w}_t = E\left(w_t/\widetilde{y}^{i}\right) = \begin{pmatrix} \sigma^2_t P^{-1}_{t/t-1} & -\sigma^2_t K'_t \\ 0 & g^{-1}_{s^i_t} \end{pmatrix} \begin{pmatrix} e^i_t \\ r_t \end{pmatrix}$$

where

$$r_{t-1} = P^{-1}_{t/t-1} e^i_t + \left(m^i_t\left(\gamma, c, \rho_1, \rho_2\right) - K_t\right) r_t$$

with $r_{T_i} \equiv 0$.

Step 3: Compute $\widehat{\varepsilon}^{i+} = E\left(\varepsilon^{i+}/\widetilde{y}^{i+}, \left\{s^i_t\right\}^{T_i}_{t=t_i}\right)$ and $\widehat{\varepsilon}^{i} = E\left(\varepsilon^{i}/\widetilde{y}^{i}, \left\{s^i_t\right\}^{T_i}_{t=t_i}\right)$ with the forwards recursion:

$$\widehat{\varepsilon}^{i}_{h(t)+1,t+1} = m^i_t\left(\gamma, c, \rho_1, \rho_2\right) \widehat{\varepsilon}^{i}_{h(t),t} + g^{-1}_{s^i_t} r_t$$

Step 4: Keep $\widetilde{\varepsilon}^{i} = \varepsilon^{i+} - \widehat{\varepsilon}^{i+} + \widehat{\varepsilon}^{i}$.

Finally,

$$\left\{\widetilde{\varepsilon}^{i}\right\}^{I}_{i=1} \sim p\left(\left\{\varepsilon^{i}_{h(t),t}\right\}^{I,T_i}_{i=1,t=t_i} / \ldots\right)$$

3. $\underline{\left(\beta^i / \left\{x^i_{h(t),t}\right\}_{i,t}, \left\{\sigma^2_t\right\}^{T}_{t=1}, \left\{\varepsilon^{i}_{h(t),t}\right\}^{I,T_i}_{i=1,t=t_i}, \mu, H, H_\alpha, y, y^*\right)'}$

Before we proceed, we need to define some objects. Let $\beta = \left(\widetilde{\beta}_0, \widetilde{\beta}_1, \ldots, \beta_k\right)'$, $\widetilde{\alpha}^i = \left(\alpha^i_0, \alpha^i_1\right)'$, and $\overline{h}^i_t = \left(1, h^i\left(t\right)\right)$; let $\widetilde{y}^i_{h(t),t} = y^i_{h(t),t} - \overline{h}^i_t \widetilde{\alpha}^i$.

(a) $p\left(\beta / \left\{x^i_{h(t),t}\right\}_{i,t}, \left\{\sigma^2_t\right\}^{T}_{t=1}, \left\{\widetilde{\alpha}^i\right\}^{N}_{i=1}, \left\{\varepsilon^{i}_{h(t),t}\right\}^{I,\,T_i}_{i=1,t=t_i}, \mu, H, y, y^*\right) \propto$

$$\propto \left[\prod_{i=1}^{I}\prod_{t=t_i}^{T_i} p\left(y^i_{h(t),t}/x^i_{h(t),t},\left\{\widetilde{\alpha}^i\right\}_{i=1}^N,\beta,\sigma_t^2,\varepsilon^i_{h(t),t}\right)\right] p\left(\beta/\mu,H\right)$$

$$\propto \left\{\prod_{i=1}^{I}\prod_{t=t_i}^{T_i} \exp\left[-\tfrac{1}{2\sigma_t^2}\left(\widetilde{y}^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta-\varepsilon^i_{h(t),t}\right)^2\right]\right\}\exp\left\{-\tfrac{1}{2}\left(\beta-\mu\right)' H\left(\beta-\mu\right)\right\}$$

$$\propto \exp\left[-\tfrac{1}{2}\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\left(\frac{\widetilde{y}^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta-\varepsilon^i_{h(t),t}}{\sigma_t}\right)^2\right]\exp\left\{-\tfrac{1}{2}\left(\beta-\mu\right)' H\left(\beta-\mu\right)\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\beta'\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\frac{x^i_{h(t),t}x^{i\prime}_{h,t}}{\sigma_t^2}\beta-2\beta'\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\frac{x^i_{h(t),t}\left(\widetilde{y}^i_{h(t),t}-\varepsilon^i_{h(t),t}\right)}{\sigma_t^2}+\beta' H\beta-2\beta' H\mu\right]\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\beta'\left(\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\frac{x^i_{h(t),t}x^{i\prime}_{h(t),t}}{\sigma_t^2}+H\right)\beta-2\beta'\left(\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\frac{x^i_{h(t),t}\left(\widetilde{y}^i_{h(t),t}-\varepsilon^i_{h(t),t}\right)}{\sigma_t^2}+H\mu\right)\right]\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\left(\beta-\overline{\mu}_\beta\right)'\overline{H}_\beta\left(\beta-\overline{\mu}_\beta\right)\right]\right\}$$

where $\overline{H}_\beta=\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\frac{x^i_{h(t),t}x^{i\prime}_{h(t),t}}{\sigma_t^2}+H$ and $\overline{\mu}_\beta=\overline{H}_\beta^{-1}\left(\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\frac{x^i_{h(t),t}\left(\widetilde{y}^i_{h(t),t}-\varepsilon^i_{h(t),t}\right)}{\sigma_t^2}+H\mu\right)$.

Thus,

$$\beta/\left(\left\{x^i_{h(t),t}\right\}_{i,t},\left\{\widetilde{\alpha}^i\right\}_{i=1}^N,\left\{\sigma_t^2\right\}_{t=1}^T,\left\{\varepsilon^i_{h(t),t}\right\}_{i=1,\,t=t_i}^{I,\,T_i},\mu,H,y,y^*\right)\sim N\left(\overline{\mu}_\beta,\overline{H}_\beta^{-1}\right).$$

(b) $p\left(\widetilde{\alpha}^i/\left\{x^i_{h(t),t}\right\}_{i,t},\beta,\left\{\sigma_t^2\right\}_{t=1}^T,\left\{\varepsilon^i_{h(t),t}\right\}_{i=1,t=t_i}^{I,\,T_i},\mu,H_\alpha,y,y^*\right)\propto$

$$\propto \left[\prod_{t=t_i}^{T_i} p\left(y^i_{h(t),t}/x^i_{h(t),t},\beta^i,\sigma_t^2,\varepsilon^i_{h(t),t}\right)\right] p\left(\widetilde{\alpha}^i/H_\alpha\right)$$

$$\propto \left\{\prod_{t=t_i}^{T_i}\exp\left[-\tfrac{1}{2\sigma_t^2}\left(y^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta^i-\varepsilon^i_{h(t),t}\right)^2\right]\right\}\exp\left\{-\tfrac{1}{2}\widetilde{\alpha}^{i\prime}H_\alpha\widetilde{\alpha}^i\right\}$$

$$\propto \exp\left[-\tfrac{1}{2}\sum_{t=t_i}^{T_i}\left(\frac{y^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta^i-\varepsilon^i_{h(t),t}}{\sigma_t}\right)^2\right]\exp\left\{-\tfrac{1}{2}\widetilde{\alpha}^{i\prime}H_\alpha\widetilde{\alpha}^i\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\widetilde{\alpha}^{i\prime}\sum_{t=t_i}^{T_i}\frac{\overline{h}_t^{i\prime}\overline{h}_t^i}{\sigma_t^2}\widetilde{\alpha}^i-2\widetilde{\alpha}^{i\prime}\sum_{t=t_i}^{T_i}\frac{\overline{h}_t^{i\prime}\left(y^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta-\varepsilon^i_{h(t),t}\right)}{\sigma_t^2}+\widetilde{\alpha}^{i\prime}H_\alpha\widetilde{\alpha}^i\right]\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\widetilde{\alpha}^{i\prime}\left(\sum_{t=t_i}^{T_i}\frac{\overline{h}_t^{i\prime}\overline{h}_t^i}{\sigma_t^2}+H_\alpha\right)\widetilde{\alpha}^i-2\widetilde{\alpha}^{i\prime}\sum_{t=t_i}^{T_i}\frac{\overline{h}_t^{i\prime}\left(y^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta-\varepsilon^i_{h(t),t}\right)}{\sigma_t^2}\right]\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\left(\widetilde{\alpha}^i-\overline{\mu}_{\widetilde{\alpha}^i}\right)'\overline{H}_{\widetilde{\alpha}^i}\left(\widetilde{\alpha}^i-\overline{\mu}_{\widetilde{\alpha}^i}\right)\right]\right\}$$

where $\overline{H}_{\widetilde{\alpha}^i}=\sum_{t=t_i}^{T_i}\frac{\overline{h}_t^{i\prime}\overline{h}_t^i}{\sigma_t^2}+H_\alpha$, and $\overline{\mu}_{\widetilde{\alpha}^i}=\overline{H}_{\widetilde{\alpha}^i}^{-1}\left(\sum_{t=t_i}^{T_i}\frac{\overline{h}_t^{i\prime}\left(y^i_{h(t),t}-x^{i\prime}_{h(t),t}\beta-\varepsilon^i_{h(t),t}\right)}{\sigma_t^2}\right)$.

Thus,

$$\widetilde{\alpha}^i/\left(\left\{x^i_{h(t),t}\right\}_{i,t},\beta,\left\{\sigma_t^2\right\}_{t=1}^T,\left\{\varepsilon^i_{h(t),t}\right\}_{i=1,\,t=t_i}^{I,\,T_i},\mu,H_\alpha,y,y^*\right)\sim N\left(\overline{\mu}_{\widetilde{\alpha}^i},\overline{H}_{\widetilde{\alpha}^i}^{-1}\right).$$

4. $$\frac{\left(\left\{\sigma_t^2\right\}_{t=1}^{T} / \left\{x_{h(t),t}^i\right\}_{i,t}, \left\{\beta^i\right\}_{i=1}^N, \left\{\varepsilon_{h(t),t}^i\right\}_{i=1,t=t_i}^{I,T_i}, y, y^*\right)'}{}$$

$$p\left(\sigma_t^2 / \left\{x_{h(t),t}^i\right\}_i, \left\{\beta^i\right\}_{i=1}^N, \left\{\varepsilon_{h(t),t}^i\right\}_i, y, y^*\right) \propto$$

$$\propto \left[\prod_{i\in B_t} p\left(y_{h(t),t}^i / x_{h(t),t}^i, \left\{\beta^i\right\}_{i=1}^N, \sigma_t^2, \kappa_i, \varepsilon_{h(t),t}^i\right)\right] p\left(\sigma_t^2\right)$$

where $B_t = \left\{i \in I \ s.t.\ y_{h(t),t}^i \text{ is in the sample}\right\}$, and $n_I^t = \#B_t$

$$\propto \left(\sigma_t^2\right)^{-\frac{n_I^t}{2}} \exp\left\{-\frac{1}{2\sigma_t^2} \sum_{i\in B_t} \left(y_{h(t),t}^i - x_{h(t),t}^{i\prime}\beta^i - \varepsilon_{h(t),t}^i\right)^2\right\} \left(\sigma_t^2\right)^{-\left(\frac{\nu_\omega}{2}-1\right)} \exp\left\{-\frac{1}{2\sigma_t^2} \underline{s}_\omega^2\right\}$$

$$\propto \left(\sigma_t^2\right)^{-\left(\frac{n_I^t}{2}+\frac{\nu_\omega}{2}-1\right)} \exp\left\{-\frac{1}{\sigma_t^2}\left[\frac{\sum_{i\in B_t}\left(y_{h(t),t}^i - x_{h(t),t}^{i\prime}\beta^i - \varepsilon_{h(t),t}^i\right)^2 + \underline{s}_\omega^2}{2}\right]\right\}$$

$$\propto \left(\sigma_t^2\right)^{-\left(\frac{n_I^t}{2}+\frac{\nu_\omega}{2}-1\right)} \exp\left\{-\frac{1}{\sigma_t^2}\left[\frac{n_I^t \bar{s}_{\omega,t}^2 + \underline{s}_\omega^2}{2}\right]\right\}$$

$$\therefore \left(\sigma_t^2\right)^{-1} / \left(\left\{x_{h(t),t}^i\right\}_{i,t}, \left\{\beta^i\right\}_{i=1}^N, \ldots\right) \sim \Gamma\left(\frac{n_I^t}{2} + \frac{\nu_\omega}{2}, \frac{\sum_{i\in B_t}\left(y_{h(t),t}^i - x_{h(t),t}^{i\prime}\beta^i - \varepsilon_{h(t),t}^i\right)^2 + \underline{s}_\omega^2}{2}\right)$$

5. $$\frac{\left(\widetilde{\rho} / \left\{\varepsilon_{h(t),t}^i\right\}_{i=1,t=t_i}^{I,T_i}, \gamma, c, \lambda, \phi, \left\{\kappa_i\right\}_{i=1}^I, \left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i}, \left\{g_j\right\}_{j=1}^m\right)'}{}$$

$$p\left(\widetilde{\rho} / \left\{\varepsilon_{h(t),t}^i\right\}_{i=1,t=t_i}^{I,T_i}, \gamma, c, \lambda, \phi, \left\{\kappa_i\right\}_{i=1}^I, \left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i}, \left\{g_j\right\}_{j=1}^m\right) \propto$$

$$\propto \left\{\prod_{i=1}^I \prod_{t=t_i+1}^{T_i} p\left(\varepsilon_{h(t),t}^i / \left\{\varepsilon_{h(t),t}^i\right\}_{t_i}^{t-1}, \widetilde{\rho}, \gamma, c, \lambda, \phi, \kappa_i, \left\{s_t^i\right\}_{t=t_i}^t, \left\{g_j\right\}_{j=1}^m\right)\right\} \times p\left(\widetilde{\rho}\right)$$

$$\propto \left\{\prod_{i=1}^I \prod_{t=t_i+1}^{T_i} p\left(\varepsilon_{h(t),t}^i / \varepsilon_{h(t)-1,t-1}^i, \cdots\right)\right\} \times p\left(\widetilde{\rho}\right)$$

$$\propto \left\{\prod_{i=1}^I \prod_{t=t_i+1}^{T_i} \exp\left\{-\frac{1}{2}\left[\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\left(\varepsilon_{h(t),t}^i - \varepsilon^i(\gamma,c)_{h(t)-1,t-1}\widetilde{\rho}\right)^2\right]\right\}\right\} \times$$

$$\times \exp\left\{-\frac{1}{2}h_\rho\left(\widetilde{\rho} - \mu_{\widetilde{\rho}}\right)'\left(\widetilde{\rho} - \mu_{\widetilde{\rho}}\right)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left\{\begin{array}{c} \sum_{i=1}^I \sum_{t=t_i+1}^{T_i} \frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\left(\varepsilon_{h(t),t}^i - \varepsilon^i(\gamma,c)_{h(t)-1,t-1}\widetilde{\rho}\right)^2 + \\ + h_\rho\left(\widetilde{\rho} - \mu_{\widetilde{\rho}}\right)'\left(\widetilde{\rho} - \mu_{\widetilde{\rho}}\right) \end{array}\right\}\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left\{\begin{array}{c} \sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\left(\begin{array}{c}\widetilde{\rho}'\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i(\gamma,c)_{h(t)-1,t-1}\,\widetilde{\rho}- \\ -2\widetilde{\rho}'\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i_{h(t),t}\end{array}\right) \\ +h_\rho\left(\widetilde{\rho}'\widetilde{\rho}-2\widetilde{\rho}'\mu_{\widetilde{\rho}}\right)\end{array}\right\}\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left\{\begin{array}{c} \widetilde{\rho}'\left(\sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i(\gamma,c)_{h(t)-1,t-1}\right)\widetilde{\rho}- \\ -2\widetilde{\rho}'\left(\sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i_{h(t),t}\right)+h_\rho\left(\widetilde{\rho}'\widetilde{\rho}-2\widetilde{\rho}'\mu_{\widetilde{\rho}}\right)\end{array}\right\}\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left\{\begin{array}{c} \widetilde{\rho}'\left(\sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i(\gamma,c)_{h(t)-1,t-1}+h_\rho I\right)\widetilde{\rho}- \\ -2\widetilde{\rho}'\left(\sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i_{h(t),t}+h_\rho\mu_{\widetilde{\rho}}\right)\end{array}\right\}\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\left(\widetilde{\rho}-\overline{\mu}_{\widetilde{\rho}}\right)'\overline{H}_{\widetilde{\rho}}\left(\widetilde{\rho}-\overline{\mu}_{\widetilde{\rho}}\right)\right]\right\}$$

where $\overline{H}_{\widetilde{\rho}}=\sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i(\gamma,c)_{h(t)-1,t-1}+h_\rho I,$ and

$$\overline{\mu}_{\widetilde{\rho}}=\overline{H}_{\widetilde{\rho}}^{-1}\left[\sum_{i=1}^{I}\sum_{t=t_i+1}^{T_i}\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\varepsilon^i(\gamma,c)'_{h(t)-1,t-1}\,\varepsilon^i_{h(t),t}+h_\rho\underline{\mu}_{\widetilde{\rho}}\right]$$

$$\therefore\ \widetilde{\rho}/\ldots\sim N\left(\overline{\mu}_{\widetilde{\rho}},\overline{H}_{\widetilde{\rho}}^{-1}\right)$$

6. $\underline{\left(c/\gamma,\phi,\left\{\varepsilon^i_{h(t),t}\right\}_{i=1,t=t_i}^{I,T_i},(\rho_1,\rho_2),\lambda,\{\kappa_i\}_{i=1}^{I},\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i},\{g_j\}_{j=1}^{m}\right)'}$

Let $A=\{i:\text{individual i has a first-age period observation}\}$, and $q=\#A$.

$$p\left(c/\gamma,\phi,\left\{\varepsilon^i_{h(t),t}\right\}_{i=1,t=t_i}^{I,T_i},(\rho_1,\rho_2),\lambda,\{\kappa_i\}_{i=1}^{I},\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i},\{g_j\}_{j=1}^{m}\right)\propto$$

$$\propto\left\{\prod_{i=1}^{I}\prod_{t=t_i}^{T_i}p\left(\varepsilon^i_{h(t),t}/\left\{\varepsilon^i_{h(t),t}\right\}_{t_i}^{t-1},(\rho_1,\rho_2),\gamma,c,\lambda,\phi,\kappa_i,\mu_{\rho_1},h_{\rho_1},\left\{s_t^i\right\}_{t=t_i}^{t_i},\{g_j\}_{j=1}^{m}\right)\right\}\times$$

$$\times p(c)$$

$$\propto\left\{\prod_{i=1}^{I}\prod_{t=t_i}^{T_i}p\left(\varepsilon^i_{h(t),t}/\varepsilon^i_{h(t)-1,t-1},\cdots\right)\right\}p(\gamma)\,p(c)\,p(\phi)$$

$$\propto\left\{\prod_{i=1}^{I}\prod_{t=t_i}^{T_i}\left(\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\right)^{\frac{1}{2}}\exp\left\{-\frac{1}{2}\left[\frac{\lambda^{I(t=t_i\,\hat{}\,i\in A)}\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)}\left(\varepsilon^i_{h(t),t}-\varepsilon^i(\gamma,c)_{h(t)-1,t-1}\,\widetilde{\rho}\right)^2\right]\right\}\right\}\times$$

$$\times p(c)$$

$$\propto \left\{ \prod_{i=1}^{I} \prod_{t=t_i}^{T_i} \left(\frac{\kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)} \right)^{\frac{1}{2}} \exp\left\{ -\frac{1}{2} \left[\frac{\lambda^{I(t=t_i \hat{} i\in A)} \kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)} \left(\varepsilon_{h(t),t}^i - \varepsilon^i(\gamma,c)_{h(t)-1,t-1} \widetilde{\rho} \right)^2 \right] \right\} \right\}$$

$$\propto \left\{ \prod_{i=1}^{I} \prod_{t=t_i}^{T_i} \left(k_t^i(\gamma,c,\phi) \right)^{-\frac{1}{2}} \right\} \times$$

$$\times \exp\left\{ -\frac{1}{2} \left[\sum_{i=1}^{I} \sum_{t=t_i}^{T_i} \frac{\lambda^{I(t=t_i \hat{} i\in A)} \kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)} \left(\varepsilon_{h(t),t}^i - \varepsilon^i(\gamma,c)_{h(t)-1,t-1} \widetilde{\rho} \right)^2 \right] \right\}$$

7. $\underline{\left(\gamma/c, \phi, \left\{ \varepsilon_{h(t),t}^i \right\}_{i=1,t=t_i}^{I,T_i}, (\rho_1,\rho_2), \lambda, \{\kappa_i\}_{i=1}^{I}, \left\{ s_t^i \right\}_{i=1,t=t_i}^{I,T_i}, \{g_j\}_{j=1}^{m} \right)'}$

$$p\left(\gamma/c, \phi, \left\{ \varepsilon_{h(t),t}^i \right\}_{i=1,t=t_i}^{I,T_i}, (\rho_1,\rho_2), \lambda, \{\kappa_i\}_{i=1}^{I}, \left\{ s_t^i \right\}_{i=1,t=t_i}^{I,T_i}, \{g_j\}_{j=1}^{m} \right) \propto$$

$$\propto \left\{ \prod_{i=1}^{I} \prod_{t=t_i}^{T_i} p\left(\varepsilon_{h(t),t}^i / \left\{ \varepsilon_{h(t),t}^i \right\}_{t_i}^{t-1}, (\rho_1,\rho_2), \gamma, c, \lambda, \phi, \kappa_i, \mu_{\rho_1}, h_{\rho_1}, \left\{ s_t^i \right\}_{t=t_i}^{t_i}, \{g_j\}_{j=1}^{m} \right) \right\} \times$$

$$\times p(\gamma)$$

$$\propto \left\{ \prod_{i=1}^{I} \prod_{t=t_i}^{T_i} \left(k_t^i(\gamma,c,\phi) \right)^{-\frac{1}{2}} \right\} \times$$

$$\times \exp\left\{ -\frac{1}{2} \left[\sum_{i=1}^{I} \sum_{t=t_i}^{T_i} \frac{\lambda^{I(t=t_i \hat{} i\in A)} \kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)} \left(\varepsilon_{h(t),t}^i - \varepsilon^i(\gamma,c)_{h(t)-1,t-1} \widetilde{\rho} \right)^2 \right] \right\} \left(1+\gamma^2\right)^{-1} I(\gamma > 0)$$

8. $\underline{\left(\phi/c, \gamma, \left\{ \varepsilon_{h(t),t}^i \right\}_{i=1,t=t_i}^{I,T_i}, (\rho_1,\rho_2), \lambda, \{\kappa_i\}_{i=1}^{I}, \left\{ s_t^i \right\}_{i=1,t=t_i}^{I,T_i}, \{g_j\}_{j=1}^{m} \right)'}$

$$p\left(\phi/c, \gamma, \left\{ \varepsilon_{h(t),t}^i \right\}_{i=1,t=t_i}^{I,T_i}, (\rho_1,\rho_2), \lambda, \{\kappa_i\}_{i=1}^{I}, \left\{ s_t^i \right\}_{i=1,t=t_i}^{I,T_i}, \{g_j\}_{j=1}^{m} \right) \propto$$

$$\propto \left\{ \prod_{i=1}^{I} \prod_{t=t_i}^{T_i} p\left(\varepsilon_{h(t),t}^i / \left\{ \varepsilon_{h(t),t}^i \right\}_{t_i}^{t-1}, (\rho_1,\rho_2), \gamma, c, \lambda, \phi, \kappa_i, \mu_{\rho_1}, h_{\rho_1}, \left\{ s_t^i \right\}_{t=t_i}^{t_i}, \{g_j\}_{j=1}^{m} \right) \right\} \times$$

$$\times p(\phi)$$

$$\propto \left\{ \prod_{i=1}^{I} \prod_{t=t_i}^{T_i} \left(k_t^i(\gamma,c,\phi) \right)^{-\frac{1}{2}} \right\} \times$$

$$\times \exp\left\{ -\frac{1}{2} \left[\sum_{i=1}^{I} \sum_{t=t_i}^{T_i} \frac{\lambda^{I(t=t_i \hat{} i\in A)} \kappa_i g_{s_t^i}}{k_t^i(\gamma,c,\phi)} \left(\varepsilon_{h(t),t}^i - \varepsilon^i(\gamma,c)_{h(t)-1,t-1} \widetilde{\rho} \right)^2 \right] \right\} \frac{1}{\phi}$$

9. $\underline{\left(\{\kappa_i\}_{i=1}^{I} / \left\{ \varepsilon_{h(t),t}^i \right\}_{i=1,t=t_i}^{I,T_i}, (\rho_1,\rho_2), (\gamma,c), \lambda, \phi, \left\{ s_t^i \right\}_{i=1,t=t_i}^{I,T_i}, \{g_j\}_{j=1}^{m} \right)'}$

$$p\left(\kappa_i / \left\{ \varepsilon_{h(t),t}^i \right\}_{t=t_i}^{T_i}, (\rho_1,\rho_2), (\gamma,c), \lambda, \phi, \left\{ s_t^i \right\}_{t=t_i}^{T_i}, \{g_j\}_{j=1}^{m} \right) \propto$$

$$\propto \left\{ \prod_{t=t_i}^{T_i} p\left(\varepsilon_{h(t),t}^i / \left\{ \varepsilon_{h(t),t}^i \right\}_{t_i}^{t-1}, ((\rho_1,\rho_2), \gamma, c, \lambda, \phi, \kappa_i, \mu_{\rho_1}, h_{\rho_1}, s_t^i, \{g_j\}_{j=1}^{m} \right) \right\} p(\kappa_i)$$

$$\propto \left\{ \prod_{t=t_i}^{T_i} p\left(\varepsilon^i_{h(t),t} / \varepsilon^i_{h(t)-1,t-1}, \cdots\right) \right\} p(\kappa_i)$$

$$\propto (\kappa_i)^{\frac{T_i}{2}} \exp\left\{ -\frac{1}{2} \sum_{t=t_i}^{T_i} \frac{\lambda^{I(t=t_i \hat{} i\in A)} \kappa_i g_{s^i_t}}{k^i_t(\gamma,c,\phi)} \left(\varepsilon^i_{h(t),t} - \varepsilon^i(\gamma,c)_{h(t)-1,t-1}\,\widetilde{\rho}\right)^2 \right\} \times$$

$$\times (\kappa_i)^{\frac{\nu_\kappa}{2}-1} \exp\left\{-\frac{1}{2} s^2_\kappa \kappa_i\right\}$$

$$\propto (\kappa_i)^{\frac{T_i+\nu_\kappa}{2}-1} \exp\left\{ -\kappa_i \frac{1}{2} \left[\sum_{t=t_i}^{T_i} \frac{\lambda^{I(t=t_i \hat{} i\in A)} g_{s^i_t}}{k^i_t(\gamma,c,\phi)} \left(\varepsilon^i_{h(t),t} - \varepsilon^i(\gamma,c)_{h(t)-1,t-1}\,\widetilde{\rho}\right)^2 + s^2_\kappa \right] \right\}$$

$$\therefore \kappa_i / \ldots \sim \Gamma\left(\frac{T_i+\underline{\nu}_\kappa}{2}, \frac{1}{2} \left[\sum_{t=t_i}^{T_i} \frac{\lambda^{I(t=t_i \hat{} i\in A)} g_{s^i_t}}{k^i_t(\gamma,c,\phi)} \left(\varepsilon^i_{h(t),t} - \varepsilon^i(\gamma,c)_{h(t)-1,t-1}\,\widetilde{\rho}\right)^2 + s^2_\kappa \right] \right)$$

10. $\underline{\left(\lambda / \left\{\varepsilon^i_{1,t_i}\right\}_{i\in A}, \{\kappa_i\}_{i\in A}, \left\{s^i_{t_i}\right\}_{i\in A}, \{g_j\}_{j=1}^m, \gamma, c, \phi\right)'}$

$$p\left(\lambda / \left\{\varepsilon^i_{1,t_i}\right\}_{i\in A}, \{\kappa_i\}_{i\in A}, \left\{s^i_{t_i}\right\}_{i\in A}, \{g_j\}_{j=1}^m, \gamma, c, \phi\right) \propto$$

$$\propto \prod_{i\in A} p\left(\varepsilon^i_{1,t_i} / \lambda, \kappa_i, s^i_{t_i}, \{g_j\}_{j=1}^m, \gamma, c, \phi\right) p(\lambda)$$

$$\propto \left\{ \prod_{i\in A} \left(\lambda \kappa_i g_{s^i_{t_i}} k^{i-1}_t(\gamma,c,\phi)\right)^{\frac{1}{2}} \exp\left\{ -\frac{1}{2} \lambda \kappa_i g_{s^i_{t_i}} k^{i-1}_t(\gamma,c,\phi) \left(\varepsilon^i_{1,t_i}\right)^2 \right\} \right\} \times$$

$$\times (\lambda)^{\frac{\nu_\lambda}{2}-1} \exp\left\{-\frac{1}{2} s^2_\lambda \lambda\right\}$$

$$\propto (\lambda)^{\frac{\nu_\lambda+q}{2}-1} \exp\left\{ -\frac{1}{2} \lambda \left[\sum_{i\in A} \kappa_i g_{s^i_{t_i}} k^{i-1}_t(\gamma,c,\phi) \left(\varepsilon^i_{1,t_i}\right)^2 + s^2_\lambda \right] \right\}$$

$$\therefore \lambda / \ldots \sim \Gamma\left(\frac{\nu_\lambda+q}{2}, \frac{1}{2} \left[\sum_{i\in A} \kappa_i g_{s^i_{t_i}} k^{i-1}_t(\gamma,c,\phi) \left(\varepsilon^i_{1,t_i}\right)^2 + s^2_\lambda \right] \right)$$

11. $\underline{(\mu/\beta, H)'}$

$$p(\mu/\beta, H) \propto \{p(\beta/\mu, H)\}\, p(\mu)$$

$$\propto \exp\left\{-\frac{1}{2}(\beta-\mu)' H (\beta-\mu)\right\} \exp\left\{ -\frac{1}{2} \left(\mu - \underline{\mu}_\mu\right)' \underline{H}_\mu \left(\mu - \underline{\mu}_\mu\right) \right\}$$

$$\propto \exp\left\{ -\frac{1}{2} \left[\mu' H \mu - 2\mu' H \beta + \mu' \underline{H}_\mu \mu - 2\mu' \underline{H}_\mu \underline{\mu}_\mu \right] \right\}$$

$$\propto \exp\left\{ -\frac{1}{2} \left[\mu' \left(H + \underline{H}_\mu\right) \mu - 2\mu' \left(H\beta + \underline{H}_\mu \underline{\mu}_\mu\right) \right] \right\}$$

$$\propto \exp\left\{ -\frac{1}{2} \left[(\mu - \overline{\mu})' \overline{H} (\mu - \overline{\mu}) \right] \right\}$$

where $\overline{H} = \left(H + \underline{H}_\mu\right)$ and $\overline{\mu} = \overline{H}^{-1} \left(H\beta + \underline{H}_\mu \underline{\mu}_\mu\right)$

$$\therefore \mu / (\beta, H) \sim N\left(\overline{\mu}, \overline{H}^{-1}\right)$$

12. $\underline{\left(H, H_\alpha / \left\{\beta^i\right\}_{i=1}^N, \mu\right)'}$

(a) $p\,(H/\beta,\mu) \propto p\,(\beta/\mu, H)\,p\,(H)$

$$\propto |H|^{\frac{1}{2}} \exp\left\{-\tfrac{1}{2}\,(\beta-\mu)'\,H\,(\beta-\mu)\right\} |H|^{\frac{\underline{\nu}_H-(k+1)-1}{2}} \exp\left\{-\tfrac{1}{2} tr\left(\underline{S}_H^{-1} H\right)\right\}$$

$$\propto |H|^{\frac{1+\underline{\nu}_H-(k+1)-1}{2}} \exp\left\{-\tfrac{1}{2} tr\left[\left((\beta-\mu)(\beta-\mu)' + \underline{S}_H^{-1}\right) H\right]\right\}$$

$$\therefore H/(\beta,\mu) \sim Wishart\left(1+\underline{\nu}_H, \left[(\beta-\mu)(\beta-\mu)' + \underline{S}_H^{-1}\right]^{-1}\right)$$

(b) $p\left(H_\alpha / \left\{\widetilde{\alpha}^i\right\}_{i=1}^N\right) \propto \left\{\prod_{i=1}^N p\left(\widetilde{\alpha}^i / H_\alpha\right)\right\} p\,(H_\alpha)$

$$\propto |H_\alpha|^{\frac{N}{2}} \exp\left\{-\tfrac{1}{2}\sum_{i=1}^N \left(\widetilde{\alpha}^i\right)' H \left(\widetilde{\alpha}^i\right)\right\} |H_\alpha|^{\frac{\underline{\nu}_\alpha-(2+1)-1}{2}} \exp\left\{-\tfrac{1}{2} tr\left(\underline{S}_\alpha^{-1} H_\alpha\right)\right\}$$

$$\propto |H_\alpha|^{\frac{N+\underline{\nu}_\alpha-(2+1)-1}{2}} \exp\left\{-\tfrac{1}{2} tr\left[\left(\sum_{i=1}^N \left(\widetilde{\alpha}^i\right)\left(\widetilde{\alpha}^i\right)' + \underline{S}_\alpha^{-1}\right) H_\alpha\right]\right\}$$

$$\therefore H_\alpha / \left(\left\{\widetilde{\alpha}^i\right\}_{i=1}^N\right) \sim Wishart\left(N+\underline{\nu}_\alpha, \left[\sum_{i=1}^N \left(\widetilde{\alpha}^i\right)\left(\widetilde{\alpha}^i\right)' + \underline{S}_\alpha^{-1}\right]^{-1}\right)$$

13. $\underline{\left(\mathbf{p} / \left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i}\right)'}$

$$p\left(\mathbf{p} / \left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i}\right) \propto \left\{\prod_{i=1}^I \prod_{t=t_i}^{T_i} p\left(s_t^i / \mathbf{p}\right)\right\} p\,(\mathbf{p})$$

$$\propto \left\{\prod_{i=1}^I \prod_{t=t_i}^{T_i} p\left(s_t^i / \mathbf{p}\right)\right\} p\,(\mathbf{p})$$

$$\propto \left\{\prod_{i=1}^I \prod_{t=t_i}^{T_i} \prod_{j=1}^m p_j^{I\left(s_t^i=j\right)}\right\} \prod_{j=1}^m p_j^{\alpha_j-1}$$

$$\propto \prod_{j=1}^m p_j^{\sum_{i=1}^I \sum_{t=t_i}^{T_i} I\left(s_t^i=j\right)+\alpha_j-1}$$

$$\therefore \mathbf{p} / \left(\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i}\right) \sim Dir\left[\left(\sum_{i=1}^I \sum_{t=t_i}^{T_i} I\left(s_t^i=j\right)+\alpha_j\right)_{j=1}^m\right]$$

14. $\underline{\left(\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i} / \left\{\varepsilon_{h(t),t}^i\right\}_{i=1,t=t_i}^{I,T_i}, \rho_1, \rho_2, \gamma, c, \lambda, \phi, \left\{\kappa_i\right\}_{i=1}^I, \left\{g_j\right\}_{j=1}^m, \mathbf{p}\right)'}$

$$p\left(s_t^i = j / \left\{\varepsilon_{h(t),t}^i\right\}_{t=t_i}^{T_i}, \rho_1, \rho_2, \gamma, c, \lambda, \phi, \kappa_i, \left\{g_j\right\}_{j=1}^m, \mathbf{p}\right) \propto$$

$$\propto p\left(\eta_t^i / s_t^i = j, \cdots\right) p\left(s_t^i = j/\mathbf{p}\right)$$

$$\underbrace{\propto \lambda^{\frac{1}{2}I(t=t_i \hat{} i\in A)}\left(\frac{\kappa_i g_j}{k_t^i(\gamma,c,\phi)}\right)^{\frac{1}{2}}\exp\left\{-\frac{1}{2}g_j\kappa_i\left(\begin{array}{c}\left(\lambda\left(\eta_{t_i}^i\right)^2\right)I\left(t=t_i \hat{} i\in A\right)+\\ +\left(1-I\left(t=t_i \hat{} i\in A\right)\right)\frac{\left(\eta_t^i\right)^2}{k_t^i(\gamma,c,\phi).}\end{array}\right)\right\}p(g_j)}_{w_j^{i,t}}$$

$$\therefore p\left(s_t^i=j/\left\{\varepsilon_{h(t),t}^i\right\}_{t=t_i}^{T_i},\rho_1,\rho_2,\gamma,c,\lambda,\phi,\{\kappa_i\}_{i=1}^I,\{g_j\}_{j=1}^m,\mathbf{p}\right)=\frac{w_j^{i,t}}{\sum\limits_{j=1}^m w_j^{i,t}}$$

15. $\underline{\left(\{g_j\}_{j=1}^m/\left\{\varepsilon_{h(t),t}^i\right\}_{i=1,t=t_i}^{I,T_i},\rho_1,\rho_2,\lambda,\gamma,c,\phi,\{\kappa_i\}_{i=1}^I,\left\{s_t^i\right\}_{i=1,t=t_i}^{I,T_i}\right)'}$

Let $C_j=\left\{(i,t):s_t^i=j\right\}$ and $\widetilde{n}_j=\#C_j$.

$$p\left(g_j/\left\{\varepsilon_{h(t),t}^i\right\}_{(i,t)\in C_j},\rho_1,\rho_2,\lambda,\gamma,c,\phi,\{\kappa_i\}_{(i,t)\in C_j},\left\{s_t^i\right\}_{(i,t)\in C_j}\right)\propto$$

$$\propto\left\{\prod_{(i,t)\in C_j}p\left(\eta_t^i/s_t^i=j,\cdots\right)\right\}p\left(g_j\right)$$

$$\propto g_j^{\frac{\widetilde{n}_j}{2}}\exp\left\{-\tfrac{1}{2}g_j\left(\sum_{\substack{\{(i,t)\in C_j\cap A^c\}\\ \cup\left\{(i,t)\in C_j\cap A,\text{ with }t\geq t_i+1\right\}}}\kappa_i\frac{\left(\eta_t^i\right)^2}{k_t^i(\gamma,c,\phi).}+\frac{\lambda\kappa_i}{k_{t_i}^i(\gamma,c,\phi)}\left(\eta_{t_i}^i\right)^2I\left(i\in A\cap C_j\right)\right)\right\}$$

$$\times\left(g_j\right)^{\frac{\nu_g}{2}-1}\exp\left\{-\tfrac{1}{2}g_j\underline{s}^2\right\}$$

$$\propto\exp\left\{-\tfrac{1}{2}g_j\left[\underline{s}^2+\sum_{\substack{\{(i,t)\in C_j\cap A^c\}\\ \cup\left\{(i,t)\in C_j\cap A,\text{ with }t\geq t_i+1\right\}}}\kappa_i\frac{\left(\eta_t^i\right)^2}{k_t^i(\gamma,c,\phi).}+\frac{\lambda\kappa_i}{k_{t_i}^i(\gamma,c,\phi)}\left(\eta_{t_i}^i\right)^2I\left(i\in A\cap C_j\right)\right]\right\}\times$$

$$\times\left(g_j\right)^{\frac{\widetilde{n}_j+\nu_g}{2}-1}$$

$$\therefore g_j/\left(\left\{\varepsilon_{h(t),t}^i\right\}_{(i,t)\in C_j},\ldots\right)\sim\Gamma\left(\frac{\widetilde{n}_j+\underline{\nu}_g}{2},\quad\frac{\underline{s}^2}{2}+\frac{\sum\limits_{\substack{\{(i,t)\in C_j\cap A^c\}\\ \cup\left\{(i,t)\in C_j\cap A,\text{ with }t\geq t_i+1\right\}}}\kappa_i\frac{\left(\eta_t^i\right)^2}{k_t^i(\gamma,c,\phi).}}{2}+\atop+\frac{\lambda\kappa_i}{k_{t_i}^i(\gamma,c,\phi)}\frac{\left(\eta_{t_i}^i\right)^2I(i\in A\cap C_j)}{2}\right)$$

A2. Restricted Income Profile Process

In the case of a RIP, the regression coefficients β^i are the same for all individuals, i.e. $\beta^i \equiv \beta\, \forall i$. The changes in the posterior distribution are minimal in most of the cases: the β^i's need to be replaced by the common β and the posterior distributions for a RIP follow. Only for the blocks 3 and 12, there is slightly more work to do. Here are how these blocks' distributions look like:

3. $\underline{\left(\beta / \left\{x^i_{h(t),t}\right\}_{i,t}, \left\{\sigma^2_t\right\}^T_{t=1}, \left\{\varepsilon^i_{h(t),t}\right\}^{I,T_i}_{i=1,t=t_i}, \mu, H, y, y^*\right)'}$

$$p\left(\beta / \left\{x^i_{h(t),t}\right\}_{i,t}, \left\{\sigma^2_t\right\}^T_{t=1}, \left\{\varepsilon^i_{h(t),t}\right\}^{I,\, T_i}_{i=1,t=t_i}, \mu, H, y, y^*\right) \propto$$

$$\propto \left[\prod_{i=1}^{I} \prod_{t=t_i}^{T_i} p\left(y^i_{h(t),t} / x^i_{h(t),t}, \beta, \sigma^2_t, \varepsilon^i_{h(t),t}\right)\right] p\left(\beta / \mu, H\right)$$

$$\propto \left\{\prod_{i=1}^{I} \prod_{t=t_i}^{T_i} \exp\left[-\tfrac{1}{2\sigma^2_t}\left(y^i_{h(t),t} - x^{i\prime}_{h(t),t}\beta - \varepsilon^i_{h(t),t}\right)^2\right]\right\} \exp\left\{-\tfrac{1}{2}\left(\beta - \mu\right)' H \left(\beta - \mu\right)\right\}$$

$$\propto \exp\left[-\tfrac{1}{2}\sum_{i=1}^{I}\sum_{t=t_i}^{T_i}\left(\frac{y^i_{h(t),t} - x^{i\prime}_{h(t),t}\beta - \varepsilon^i_{h(t),t}}{\sigma_t}\right)^2\right] \exp\left\{-\tfrac{1}{2}\left(\beta - \mu\right)' H \left(\beta - \mu\right)\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\beta' \sum_{i=1}^{I}\sum_{t=t_i}^{T_i} \frac{x^i_{h(t),t} x^{i\prime}_{h,t}}{\sigma^2_t}\beta - 2\beta' \sum_{i=1}^{I}\sum_{t=t_i}^{T_i} \frac{x^i_{h(t),t}\left(y^i_{h(t),t} - \varepsilon^i_{h(t),t}\right)}{\sigma^2_t} + \beta' H \beta - 2\beta' H \mu\right]\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\beta'\left(\sum_{i=1}^{I}\sum_{t=t_i}^{T_i} \frac{x^i_{h(t),t} x^{i\prime}_{h(t),t}}{\sigma^2_t} + H\right)\beta - 2\beta'\left(\sum_{i=1}^{I}\sum_{t=t_i}^{T_i} \frac{x^i_{h(t),t}\left(y^i_{h(t),t} - \varepsilon^i_{h(t),t}\right)}{\sigma^2_t} + H\mu\right)\right]\right\}$$

$$\propto \exp\left\{-\tfrac{1}{2}\left[\left(\beta - \overline{\mu}_\beta\right)' \overline{H}_\beta \left(\beta - \overline{\mu}_\beta\right)\right]\right\}$$

where $\overline{H}_\beta = \sum_{i=1}^{I}\sum_{t=t_i}^{T_i} \frac{x^i_{h(t),t} x^{i\prime}_{h(t),t}}{\sigma^2_t} + H$ and $\overline{\mu}_\beta = \overline{H}_\beta^{-1}\left(\sum_{i=1}^{I}\sum_{t=t_i}^{T_i} \frac{x^i_{h(t),t}\left(y^i_{h(t),t} - \varepsilon^i_{h(t),t}\right)}{\sigma^2_t} + H\mu\right)$.

Thus,

$$\beta / \left(\left\{x^i_{h(t),t}\right\}_{i,t}, \left\{\sigma^2_t\right\}^T_{t=1}, \left\{\varepsilon^i_{h(t),t}\right\}^{I,\, T_i}_{i=1,\, t=t_i}, \mu, H, y, y^*\right) \sim N\left(\overline{\mu}_\beta, \overline{H}_\beta^{-1}\right).$$

12. $\underline{\left(H / \beta, \mu\right)'}$

$$p\left(H / \beta, \mu\right) \propto p\left(\beta / \mu, H\right) p\left(H\right)$$

$$\propto \left|H\right|^{\frac{1}{2}} \exp\left\{-\tfrac{1}{2}\left(\beta - \mu\right)' H \left(\beta - \mu\right)\right\} \left|H\right|^{\frac{\underline{\nu}_H - (k+1) - 1}{2}} \exp\left\{-\tfrac{1}{2} tr\left(\underline{S}_H^{-1} H\right)\right\}$$

$$\propto \left|H\right|^{\frac{1 + \underline{\nu}_H - (k+1) - 1}{2}} \exp\left\{-\tfrac{1}{2} tr\left[\left(\left(\beta - \mu\right)\left(\beta - \mu\right)' + \underline{S}_H^{-1}\right) H\right]\right\}$$

$$\therefore H / \left(\beta, \mu\right) \sim Wishart\left(1 + \underline{\nu}_H, \left[\left(\beta - \mu\right)\left(\beta - \mu\right)' + \underline{S}_H^{-1}\right]^{-1}\right)$$

Appendix B Hyperparameters

The hyperparameters are selected to ensure the prior distributions are sufficiently flexible, covering all plausible values of the parameters being studied. This approach aligns with the methodologies found in the literature, notably referenced by Heathcote et al. (2010).

The term $\omega^i_{h(t),t}$ accounts for the transitory component of an agent's productivity and measurement errors in survey data. As discussed, the variance of $\omega^i_{h(t),t}$ follows a chi-squared distribution. The chosen hyperparameters, $\nu\omega = 200$ and $s\omega^2 = 15$, set the 90% confidence interval for σ^2_t between 0.064 and 0.089, with a 99% chance of it falling between 0.02 and 0.11.

For the inverse of g_j, denoted as g_j^{-1}, the parameters $s^2 = 0.2$ and $\nu_g = 2$ suggest a 90% probability range for the variance of η^i_t from 0.033 to 1.95.

The vector $\mathbf{p}$ follows a Dirichlet distribution with parameter $\alpha = 10 \times \mathbf{1}M$, ensuring uniformity over the $M-1$ simplex and similarity among the $pj's$.

Regarding persistence, the mean of $\widetilde{\rho}$ is set to $\mu\widetilde{\rho} = (0.5, 0)'$ with a precision of $h\rho I = 0.5I$. The parameter λ, affecting the first age-period shocks' variance, follows a chi-squared distribution with $s^2_\lambda = 2$ and $\nu_\lambda = 2$. This results in a 90% probability that λ^{-1} lies between 0.33 and 19.5, reflecting a non-informative prior but allowing for significant variance adjustment.

The prior for μ is non-informative, with a mean of **0** and a dispersion controlled by $H_\mu = 0.01\mathbf{I}$. Similarly, H follows a Wishart distribution with parameters $\nu H = 10$ and $SH = 1,000\mathbf{I}$, indicating a high level of uncertainty.

Lastly, the individual-specific component κ_i, related to the variance of η^i_t, is modeled as a chi-squared variable. The chosen degrees of freedom $\nu\kappa = 2$ and scale factor $s\kappa^2 = 1$ allow κ_i^{-1} to vary significantly, with a 90% chance of it being between 0.17 and 9.75, reflecting the potential for both substantial increases and modest decreases in individual variance.

Tables and Figures.

Table 1: Wages per Hour Worked

Summary Statistics	Complete Sample	High-School Sample	College Sample
Minimum Wage:	\$2.64	\$2.65	\$2.64
25^{th} Percentile:	\$15.29	\$13.48	\$21.62
Median:	\$22.47	\$19.38	\$31.25
Mean:	\$26.51	\$21.25	\$37.79
75^{th} Percentile:	\$31.66	\$26.50	\$44.23
Maximum Wage:	\$597.33	\$415.60	\$597.33
Std. deviation:	\$22.15	\$12.65	\$33.89
Interquantile Range:	\$16.37	\$13.02	\$22.61
Number of observations:	61,163	32,808	15,445
Number of individuals:	5,398	3,161	1,339
Average number of years an individual is in the sample:	11.3	10.4	11.5

Note: Summary Statistics for the PSID Sample 1968-2011. The High-school Sample includes all the observations from the complete sample where the head of the household has at most high-school education. Instead the College Sample includes only the observations where the head has a college degree or graduate studies. All monetary values are expressed in 2007 dollars.

Table 2: Restricted Income Profile Process (RIP)

Parameters	GMM	Bayesian
ρ	**0.976** (0.0120)	**0.967** (0.0029)
σ^2_ω	**0.095** (0.0153)	**0.094** (0.0157)
σ^2_η	**0.029** (0.0103)	**0.025** (0.0009)

Note: Estimates for the autocorrelation coefficient (ρ) and the variances of the transitory and persistent shocks (σ^2_ω and σ^2_η, respectively) under different estimation strategies. Standard Errors are written in parenthesis. In the case of the GMM estimates, the variances where computed by means of a Block Bootstrap with 100 repetitions.

Table 3: Detailed Breakdown of Persistent Shock Components

Parameter	Mode	Mean	Std. Dev.	25%	50% (Median)	75%
λ^{-1}	1.034	1.034	0.0031	1.032	1.034	1.036
κ_i^{-1}	0.520	2.269	4.9093	0.481	0.693	1.728
g^{-1}	0.025	0.027	0.0009	0.026	0.027	0.029

Table 4: Models with Normal and Mixture-of-Normal Errors

Parameters	Normal Errors	Mixture-of-Normal Errors
	All Heads of the Household	
ρ	**0.967** (0.0029)	**0.992** (1.193e-03)
σ^2_ω	**0.094** (0.0157)	**0.092** (7.697e-03)
g_1^{-1}	**0.025** (0.0009)	**0.0003** (5.947e-06)
p_{g_1}	**100%** (-)	**78.5%** (3.742e-03)
g_2^{-1}	**-** (-)	**0.2574** (8.217e-03)
p_{g_2}	**-** (-)	**21.5%** (3.742e-03)
Excess Kurtosis	**0.0**	**10.8**
	Heads with High School Education	
ρ	**0.957** (0.0067)	**0.989** (3.437e-03)
σ^2_ω	**0.106** (0.0383)	**0.105** (1.7193e-02)
g_1^{-1}	**0.025** (0.0014)	**0.0005** (2.968e-05)
p_{g_1}	**100%** (-)	**79.9%** (8.599e-03)
g_2^{-1}	**-** (-)	**0.2176** (1.456e-02)
p_{g_2}	**-** (-)	**20.1%** (8.599e-03)
Excess Kurtosis	**0.0**	**11.7**
	Heads with College Education	
ρ	**0.950** (0.0080)	**0.987** (3.038e-03)
σ^2_ω	**0.094** (0.0232)	**0.090** (1.069e-02)
g_1^{-1}	**0.034** (0.0024)	**0.0014** (9.425e-05)
p_{g_1}	**100.0%** (-)	**79.4%** (9.909e-03)
g_2^{-1}	**-** (-)	**0.2035** (1.562e-02)
p_{g_2}	**-** (-)	**20.6%** (9.909e-03)
Excess Kurtosis	**0.0**	**10.8**

Note: Mode estimates for the autocorrelation coefficient (ρ), the variance of the transitory shocks (σ^2_ω) and the variance of the persistent shocks for each Normal distribution (g_i^{-1}) with its corresponding probability (p_{g_i}). "Heads with High School Education" includes heads with high school studies as wells as high-school drop-outs; "Heads with College Education" includes all heads of the household with a college degree and/or graduate studies. Standard Errors are written in parenthesis.

Table 5: 1-Regime Models' Persistent Shocks

	Model with Normal Errors						Model with Mixture of Normal Errors					
Parameters	Mode	Mean	Std. Dev.	25%	50%	75%	Mode	Mean	Std. Dev.	25%	50%	75%
All Heads of the Household												
λ^{-1}	4.213	4.219	0.2544	4.046	4.212	4.384	1.385e-03	1.435e-03	5.597e-05	1.396e-03	1.433e-03	1.471e-03
κ_i^{-1}	0.520	2.269	4.9093	0.481	0.693	1.728	0.437	2.141	35.079	0.403	0.531	0.736
g_1^{-1}	0.025	0.027	0.0009	0.026	0.027	0.029	2.504e-04	2.508e-04	5.947e-06	2.468e-04	2.507e-04	2.547e-04
p_{g_1}	100.0%	100.0%	-	100.0%	100.0%	100.0%	78.5%	78.5%	3.742e-03	78.1%	78.4%	78.7%
g_2^{-1}	-	-	-	-	-	-	0.2574	0.2579	8.217e-03	0.2522	0.2575	0.2633
Heads with High School Education												
λ^{-1}	3.800	3.829	0.3879	3.566	3.819	4.086	1.905e-02	9.136e-02	0.2860	2.541e-03	2.821e-03	1.782e-02
κ_i^{-1}	0.501	1.894	3.8184	0.500	0.676	1.403	0.444	1.666	32.0312	0.422	0.547	0.717
g_1^{-1}	0.025	0.026	1.422e-03	0.025	0.026	0.027	4.733e-04	4.611e-04	2.968e-05	4.407e-04	4.551e-04	4.739e-04
p_{g_1}	100.0%	100.0%	-	100.0%	100.0%	100.0%	79.9%	79.9%	8.599e-03	79.5%	80.0%	80.5%
g_2^{-1}	-	-	-	-	-	-	0.2176	0.2191	1.456e-02	0.2102	0.2194	0.2283
Heads with College Education												
λ^{-1}	2.340	2.407	0.4454	2.098	2.376	2.681	1.125e-02	1.187e-02	1.335e-03	1.092e-02	1.176e-02	1.271e-02
κ_i^{-1}	0.593	2.020	4.1312	0.504	0.683	1.406	0.610	1.390	7.075	0.419	0.576	0.895
g_1^{-1}	0.034	0.034	2.439e-03	0.032	0.034	0.035	1.372e-03	1.410e-03	9.425e-05	1.346e-03	1.405e-03	1.463e-03
p_{g_1}	100.0%	100.0%	-	100.0%	100.0%	100.0%	79.4%	79.2%	9.909e-03	78.5%	79.2%	79.9%
g_2^{-1}	-	-	-	-	-	-	0.2035	0.2050	1.562e-02	0.1942	0.2044	0.2153

Note: Summary statistics for the components of the persistent shocks' variance, namely the loading factor for the Age-1 variance (λ^{-1}), the individual loading factor (κ_i^{-1}), the variance for the i's regime (g_i^{-1}), and the probability of occurrence for the 1^{st} regime (p_{g_1}). "Heads with High School Education" includes heads with high school studies as wells as high-school drop-outs; "Heads with College Education" includes all heads of the household with a college degree and/or graduate studies.

Table 6: Characterization of the different Risk Groups

Variables	**Min.**	**25%**	**Median**	**Mean**	**75%**	**Max.**
	Low-Risk Individuals					
Average Age	25.5	30.5	36.3	38.5	44.7	63.5
Years in the Sample	2.0	5.0	11.0	13.6	20.0	40.0
Average Household Size (OECD)	1.0	1.6	1.9	1.9	2.1	5.4
Moving Frequency	0.0%	0.0%	0.0%	2.7%	0.0%	66.7%
Maximum Job-Tenure (years)	0.0	6.0	11.0	11.9	20.0	20.0
Frequency of Job Changes	0.0%	10.0%	30.0%	30.2%	50.0%	90.0%
Frequency of Occupational Changes	0.0%	0.0%	18.8%	22.0%	37.5%	85.7%
	Normal-Risk Individuals					
Average Age	25.5	30.8	36.5	38.5	45.4	63.5
Years in the Sample	2.0	4.3	10.0	12.6	19.8	38.0
Average Household Size (OECD)	1.0	1.5	1.8	1.8	2.1	3.3
Moving Frequency	0.0%	0.0%	0.0%	3.7%	0.0%	66.7%
Maximum Job-Tenure (years)	0.0	4.1	9.0	10.1	16.1	20.0
Frequency of Job Changes	0.0%	14.7%	33.3%	33.2%	50.0%	87.5%
Frequency of Occupational Changes	0.0%	0.0%	20.0%	23.4%	41.5%	83.3%
	High-Risk Individuals					
Average Age	25.5	33.0	38.8	40.6	47.4	63.5
Years in the Sample	2.0	5.0	11.0	13.5	20.0	40.0
Average Household Size (OECD)	1.0	1.6	1.9	1.9	2.1	4.0
Moving Frequency	0.0%	0.0%	0.0%	3.9%	0.0%	80.0%
Maximum Job-Tenure (years)	0.0	4.5	9.8	10.5	18.0	20.0
Frequency of Job Changes	0.0%	20.3%	37.5%	36.8%	50.0%	92.9%
Frequency of Occupational Changes	0.0%	0.0%	18.8%	21.9%	36.4%	85.7%

Notes: 1) Low-Risk Individuals are defined as those subjects with an individual-specific loading factor $\kappa_i^{-1/2}$ less than 0.95; Normal-Risk Individuals have a $\kappa_i^{-1/2}$ between 0.95 and 1.05; while High-Risk Individuals have a $\kappa_i^{-1/2}$ higher than 1.5. 2) Household size is defined according to the OECD-modified scale where the first adult in the household is counted as 1, and all subsequent persons older than 14 years old are counted as 0.5; all children are counted as 0.3. 3) Frequencies are computed as the percentage of the total time an individual is followed where a change in his status has occurred. 4) Moving Frequency is the percentage of the total time an individual was followed where he has moved to another state. 5) See the Appendix for an explanation of the definition used for jobs and occupations.

Table 7: Comparison between the Lowest- and Highest-Risk Groups

Variables	Min.	25%	Median	Mean	75%	Max.
	Lowest-Risk Individuals					
Average Age	28.5	36.5	40.8	42.5	49.3	60.0
Years in the Sample	8.0	16.0	22.0	22.7	29.0	40.0
Average Household Size (OECD)	1.0	1.7	2.0	2.0	2.2	5.4
Moving Frequency	0.0%	0.0%	0.0%	1.2%	0.0%	23.5%
Maximum Job-Tenure (years)	1.0	15.0	20.0	17.4	20.0	20.0
Frequency of Job Changes	0.0%	13.0%	26.7%	28.1%	40.0%	77.8%
Frequency of Occupational Changes	0.0%	7.1%	18.2%	20.9%	33.3%	75.0%
	Highest-Risk Individuals					
Average Age	25.5	34.1	39.7	42.0	50.0	63.5
Years in the Sample	2.0	5.0	11.0	12.7	19.0	38.0
Average Household Size (OECD)	1.0	1.6	1.9	1.9	2.1	3.6
Moving Frequency	0.0%	0.0%	0.0%	3.6%	0.0%	80.0%
Maximum Job-Tenure (years)	0.0	4.0	10.0	10.5	19.0	20.0
Frequency of Job Changes	0.0%	23.9%	40.0%	39.7%	55.6%	92.9%
Frequency of Occupational Changes	0.0%	0.0%	16.7%	21.3%	35.2%	85.7%

Notes: 1) Lowest-Risk Individuals are defined as those subjects with an individual-specific loading factor $\kappa_i^{-1/2}$ less than 0.6, and Highest-Risk Individuals have a $\kappa_i^{-1/2}$ higher than 2.5. 2) Household size is defined according to the OECD-modified scale where the first adult in the household is counted as 1, and all subsequent persons older than 14 years old are counted as 0.5; all children are counted as 0.3. 3) Frequencies are computed as the percentage of the total time an individual is followed where a change in his status has occurred. 4) Moving Frequency is the percentage of the total time an individual was followed where he has moved to another state. 5) See the Appendix for an explanation of the definition used for jobs and occupations.

Table 8: Persistence Parameters

Parameters	Age 25/ No Tenure	Age 35/ 2 yrs Tenure	Age 45/ 5 yrs Tenure	Age 55/ 8 yrs Tenure	Age 64/ 10 yrs Tenure
	All Heads of the Household				
1-Regime Normal Errors: ρ	0.967	0.967	0.967	0.967	0.967
1-Regime Mixture of Normal Errors: ρ	0.992	0.992	0.992	0.992	0.992
2-Regimes (Age as threshold): ρ	0.959	0.968	0.970	0.970	0.970
2-Regimes (Job-Tenure as threshold): ρ	0.840	0.939	0.992	0.992	0.992
	Heads with High School Education				
1-Regime Normal Errors: ρ	0.957	0.957	0.957	0.957	0.957
1-Regime Mixture of Normal Errors: ρ	0.989	0.989	0.989	0.989	0.989
2-Regimes (Age as threshold): ρ	0.949	0.949	0.952	0.967	0.984
2-Regimes (Job-Tenure as threshold): ρ	0.787	0.910	0.990	0.996	0.996
	Heads with College Education				
1-Regime Normal Errors: ρ	0.950	0.950	0.950	0.950	0.950
1-Regime Mixture of Normal Errors: ρ	0.987	0.987	0.987	0.987	0.987
2-Regimes (Age as threshold): ρ	0.957	0.956	0.944	0.933	0.935
2-Regimes (Job-Tenure as threshold): ρ	0.849	0.941	0.974	0.974	0.974

Note: Mode Estimates for a sample of ages in a model which allows for different ρ's in the life-cycle (2-Regimes) and a model which doesn't (1-Regime). "Heads with High School Education" includes heads with high school studies as wells as high-school drop-outs; "Heads with College Education" includes all heads of the household with a college degree and/or graduate studies.

Table 9: Weight Function Parameters.

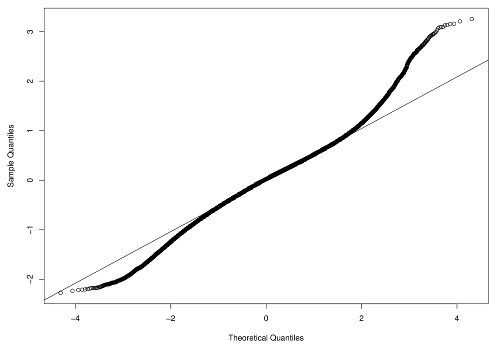
Parameters	Mode	Mean	Std. Dev.	25%	50%	75%
	All Heads of the Household					
	Age as Threshold Variable					
c	29.30	30.72	1.5575	29.39	30.68	33.48
γ	0.52	0.76	0.4873	0.44	0.59	0.89
	Job-Tenure as Threshold Variable					
c	1.032	1.034	0.0321	1.010	1.024	1.047
γ	1.044	1.045	0.0682	0.997	1.041	1.089
	Heads with High School Education					
	Age as Threshold Variable					
c	55.73	55.19	0.9010	54.94	55.48	55.78
γ	0.44	0.72	0.4863	0.37	0.54	0.94
	Job-Tenure as Threshold Variable					
c	1.055	1.062	0.0562	1.019	1.045	1.088
γ	0.931	0.935	9.7998e-02	0.866	0.925	0.999
	Heads with College Education					
	Age as Threshold Variable					
c	44.70	45.02	2.4652	44.02	45.20	46.42
γ	0.32	0.61	0.5383	0.25	0.40	0.78
	Job-Tenure as Threshold Variable					
c	1.117	1.136	0.1124	1.049	1.108	1.194
γ	1.408	1.426	0.2122	1.270	1.416	1.575

Note: Summary statistics for the threshold variable c and the smoothing parameter γ. "Heads with High School Education" includes heads with high school studies as wells as high-school drop-outs; "Heads with College Education" includes all heads of the household with a college degree and/or graduate studies.

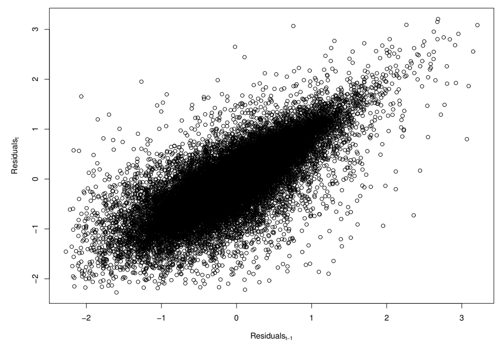
Table 10: 2-Regime Models' Persistent Shocks

Parameters	Mode	Mean	Std. Dev.	25%	50%	75%
	All Heads of the Households					
	Age as Threshold Variable					
λ^{-1}	3.519	3.531	0.2709	3.343	3.524	3.712
κ_i^{-1}	0.485	2.367	5.2797	0.479	0.692	1.750
g^{-1}	0.032	0.033	0.0023	0.031	0.033	0.035
ϕ	0.752	0.750	0.0537	0.710	0.751	0.794
	Job-Tenure as Threshold Variable					
λ^{-1}	3.131	3.134	0.1794	3.014	3.132	3.254
κ_i^{-1}	0.427	2.300	6.0194	0.476	0.694	1.649
g^{-1}	0.095	0.096	3.144e-03	0.094	0.096	0.098
ϕ	0.168	0.168	6.744e-03	0.163	0.168	0.172
	Heads with High-School Education					
	Age as Threshold Variable					
λ^{-1}	3.705	3.721	0.3804	3.457	3.712	3.971
κ_i^{-1}	0.513	1.983	4.1754	0.495	0.675	1.440
g^{-1}	0.026	0.027	1.467e-03	0.026	0.0274	0.028
ϕ	0.662	0.662	8.4647e-02	0.607	0.659	0.716
	Job-Tenure as Threshold Variable					
λ^{-1}	3.177	3.207	0.2945	3.004	3.190	3.395
κ_i^{-1}	0.523	1.932	4.6433	0.495	0.682	1.387
g^{-1}	0.079	0.081	4.317e-03	0.078	0.081	0.084
ϕ	0.191	0.191	1.201e-02	0.182	0.190	0.199
	Heads with College Education					
	Age as Threshold Variable					
λ^{-1}	2.577	2.683	0.5082	2.323	2.640	2.993
κ_i^{-1}	0.777	1.978	4.033	0.504	0.689	1.403
g^{-1}	2.969e-02	2.983e-02	2.314e-03	2.830e-02	2.977e-02	3.125e-02
ϕ	1.545	1.559	0.1593	1.442	1.539	1.664
	Job-Tenure as Threshold Variable					
λ^{-1}	1.479	1.527	0.2934	1.322	1.503	1.704
κ_i^{-1}	0.529	2.098	4.9632	0.493	0.688	1.419
g^{-1}	0.086	0.085	6.303e-03	0.081	0.085	0.089
ϕ	0.253	0.255	0.0202	0.241	0.253	0.267

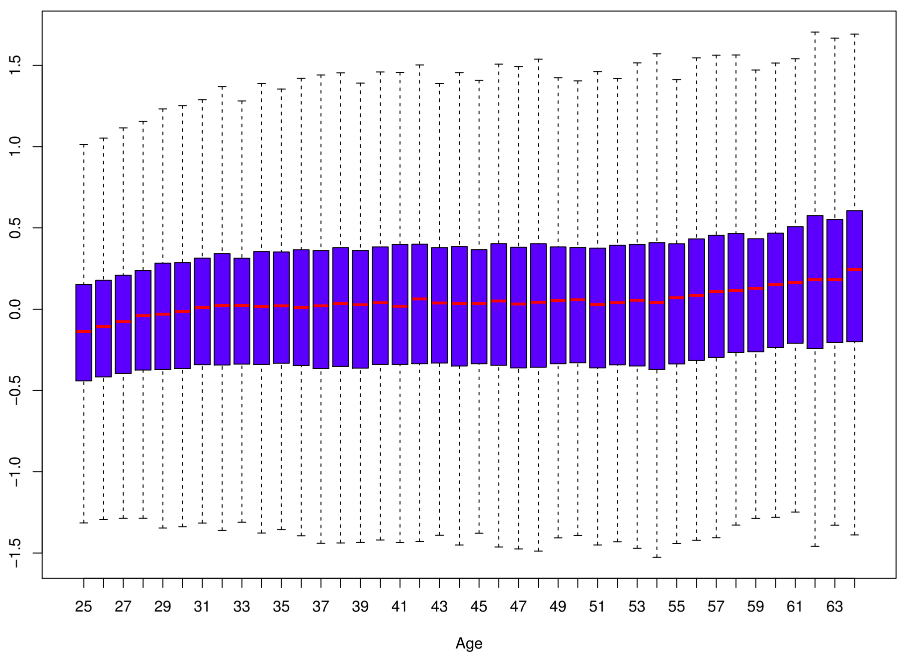
Note: Summary statistics for the components of the persistent shocks' variance, namely the loading factor for the Age-1 variance (λ^{-1}), the individual loading factor (κ_i^{-1}) and the common variance for all individuals (g^{-1}).



(a) Normal Q-Q Plot.

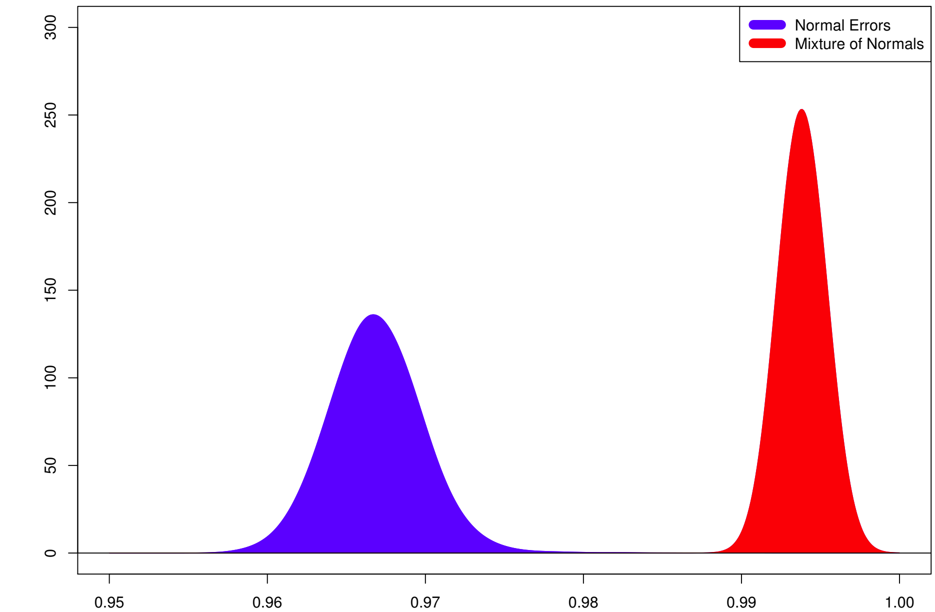


(b) Lag Plot.

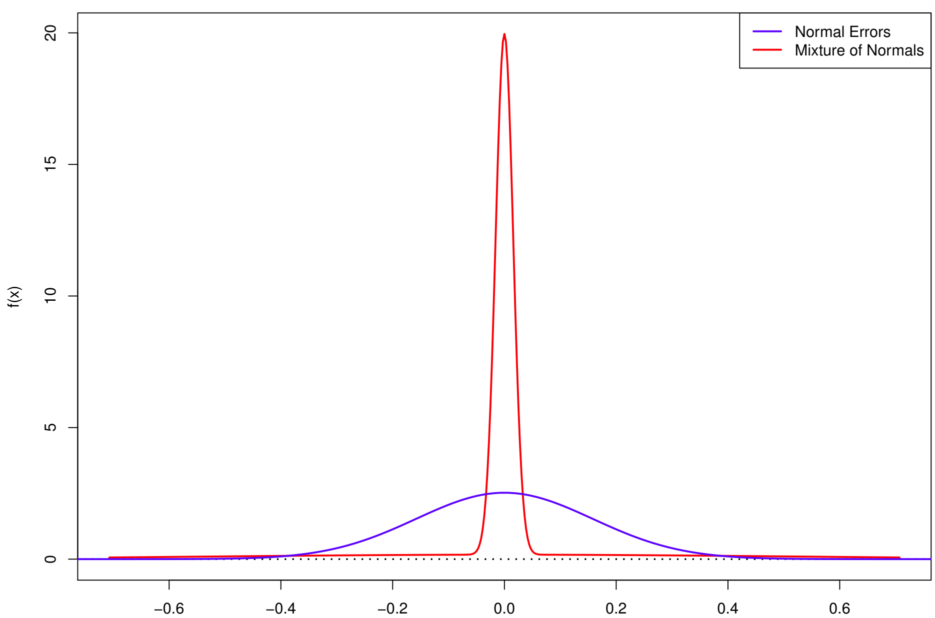


(c) Boxplots.

Figure 1: Residuals' Exploratory Analysis.

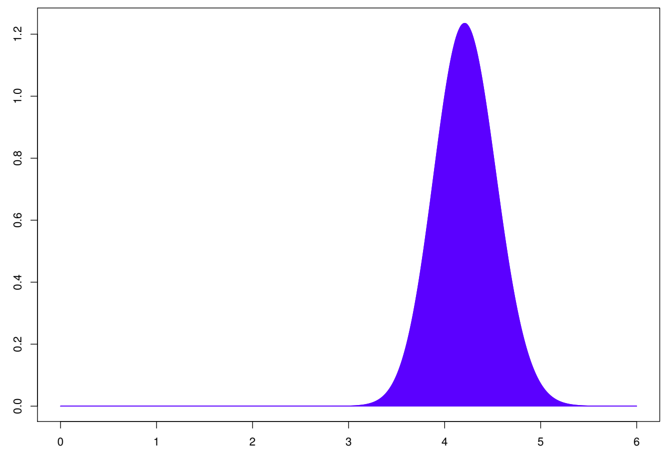


(a) Posterior Densities of the Correlation Coefficient ρ.

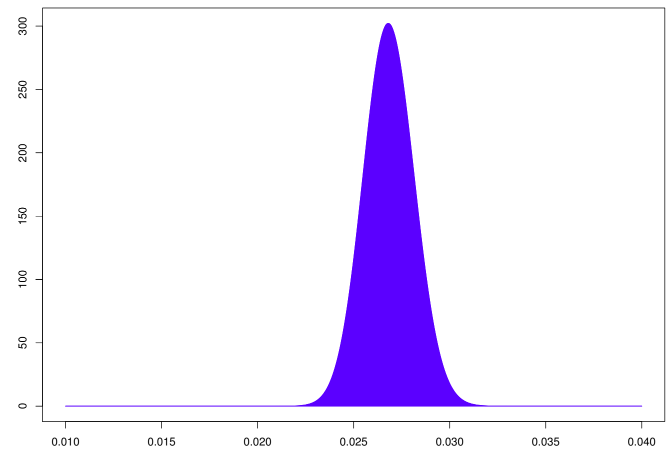


(b) Distribution of the Persistent Shocks Errors.

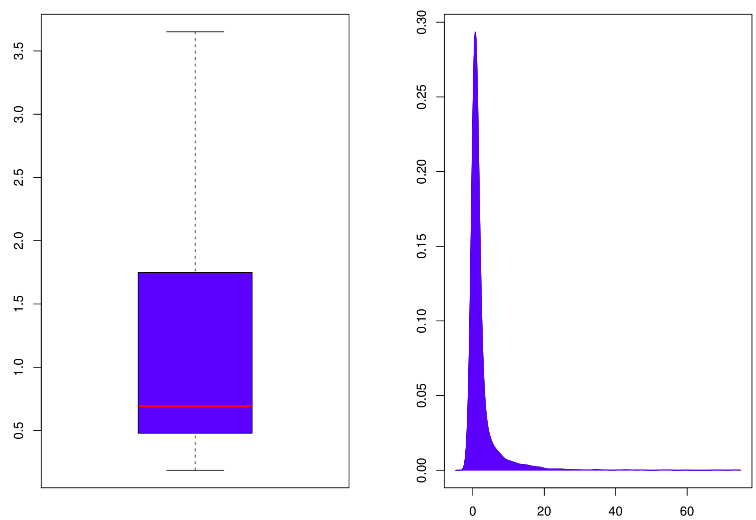
Figure 2: 1-Regime Models.



(a) Age-1-Variance Loading Factor λ^{-1}

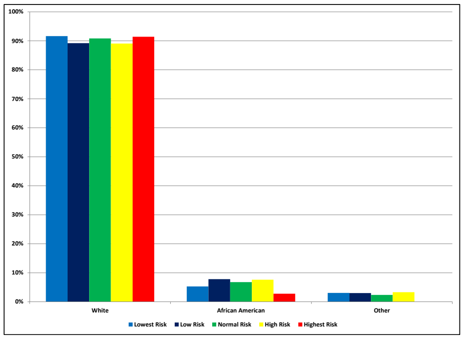


(b) Variance of the Persistent Shocks g^{-1}

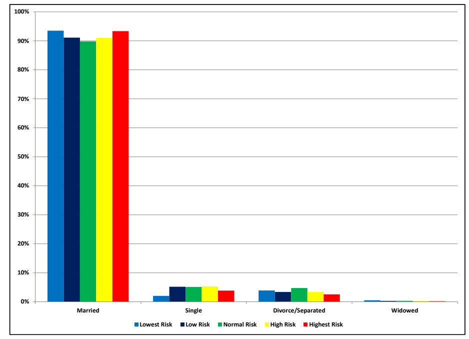


(c) Individual-Specific Loading Factor κ_i^{-1}

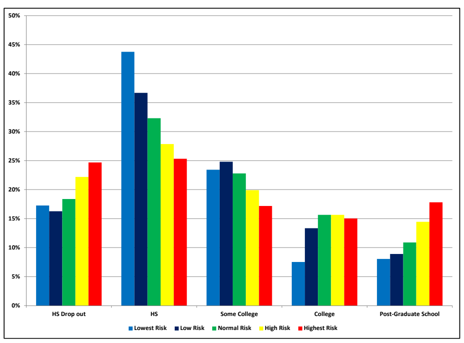
Figure 3: Posterior Densities 1-Regime Normal Errors Model



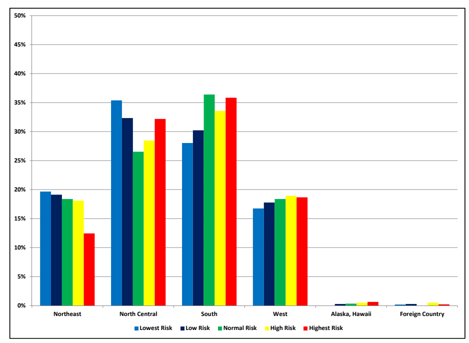
(a) Race



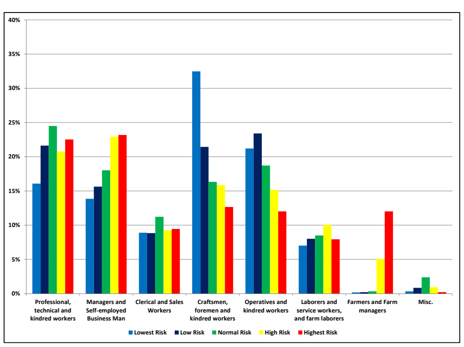
(b) Marital Status



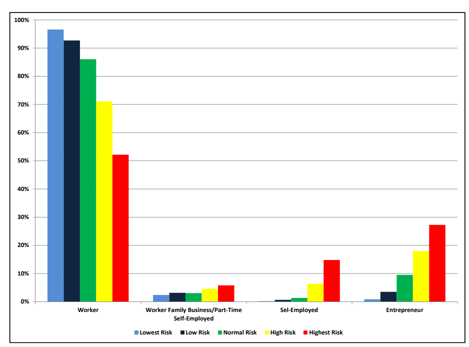
(c) Education



(d) Region

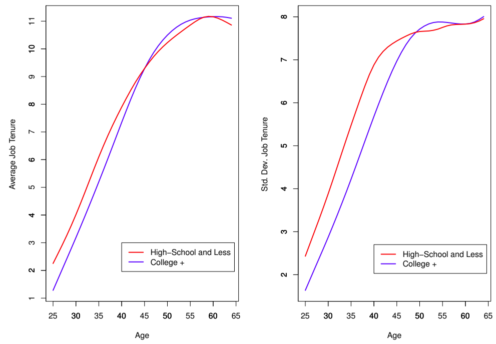


(e) Job

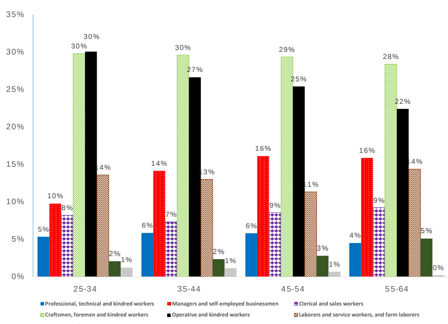


(f) Worker/Manager

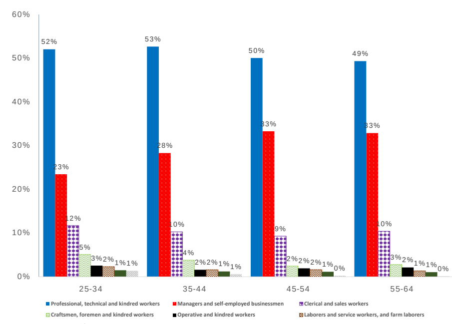
Figure 4: Analysis of Different Risk Groups



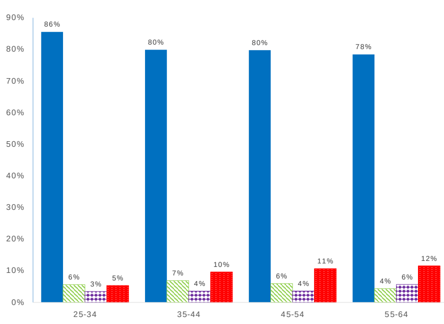
(a) Average Job Tenure and Standard Deviation



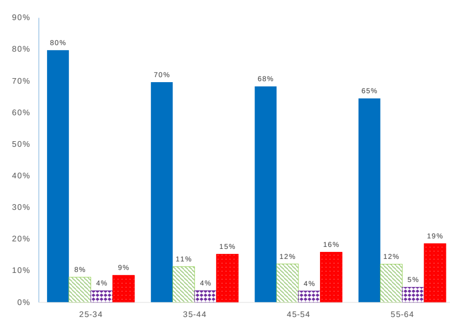
(b) Occupations done by High-School Group



(c) Occupations done by College Graduates

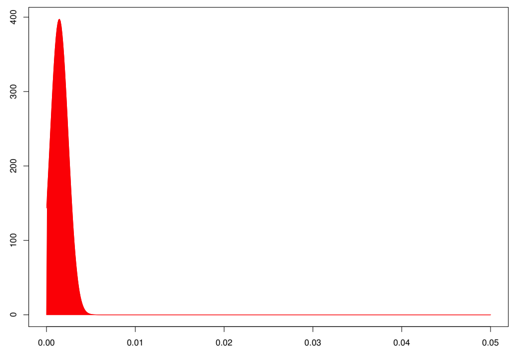


(d) Positions occupied by High-School Group

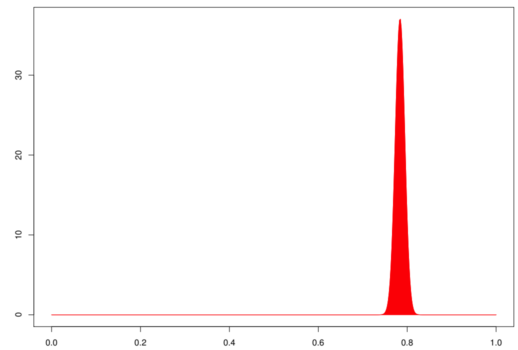


(e) Positions occupied by College Graduates

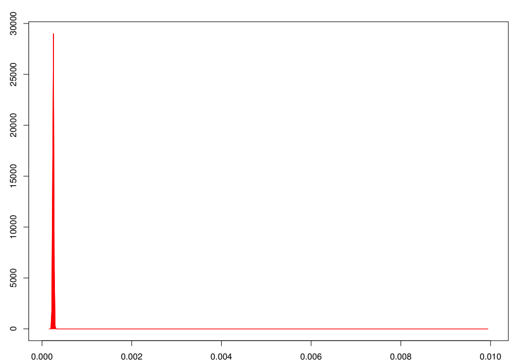
Figure 5: Profile of Jobs, Tenure and Occupations by Age and Educational Groups



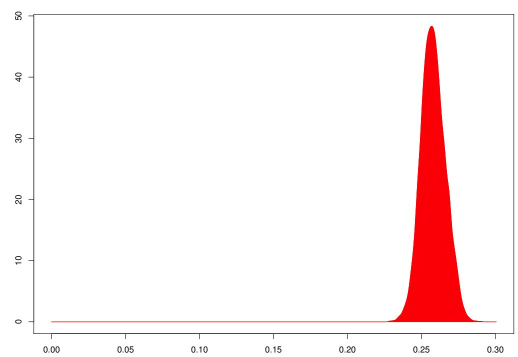
(a) Age-1-Variance Loading Factor λ^{-1}



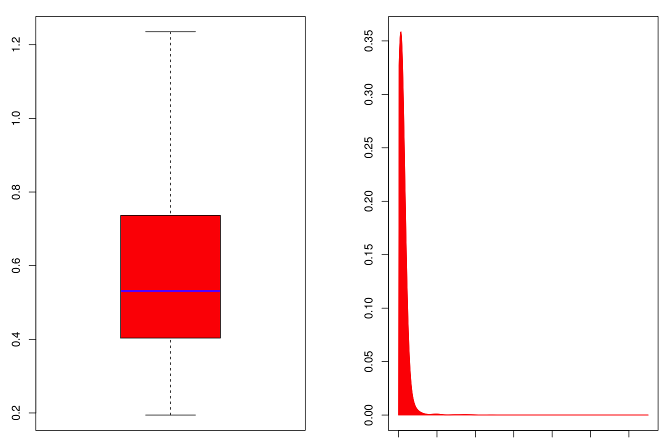
(b) First Regime's Probability p_{g_1}



(c) Variance of the Persistent Shocks g_1^{-1}

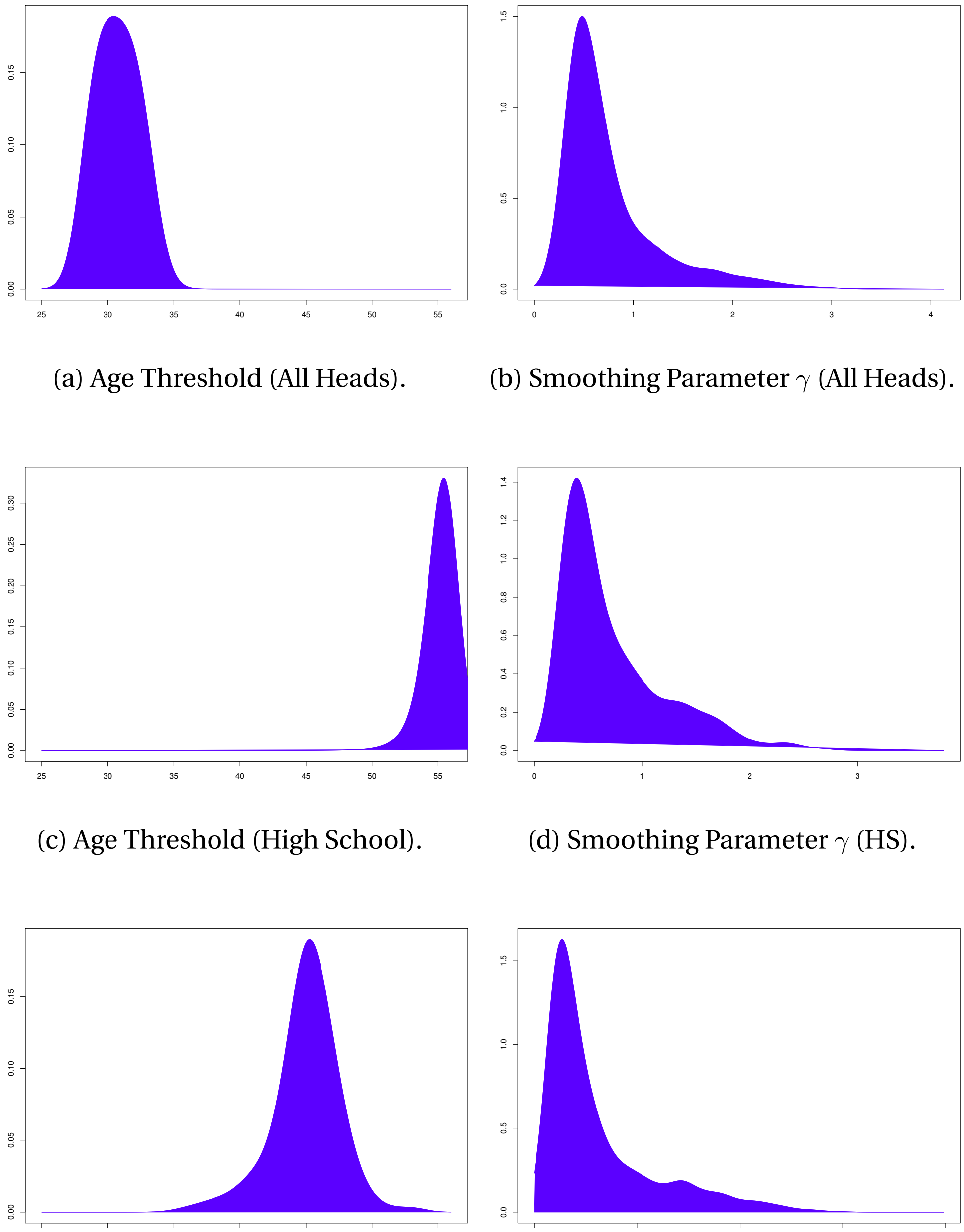


(d) Variance of the Persistent Shocks g_2^{-1}



(e) Individual-Specific Loading Factor κ_i^{-1}

Figure 6: Posterior Densities 1-Regime Mixture-of-Normal-Errors Model



(a) Age Threshold (All Heads).

(b) Smoothing Parameter γ (All Heads).

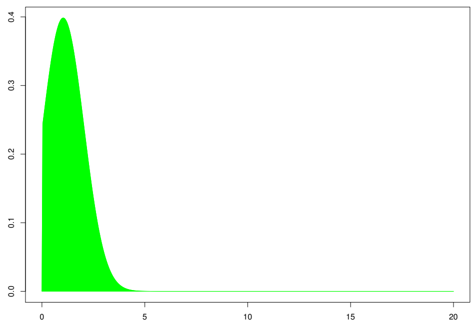
(c) Age Threshold (High School).

(d) Smoothing Parameter γ (HS).

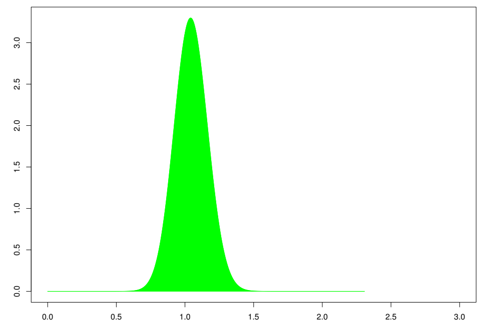
(e) Age Threshold (College).

(f) Smoothing Parameter γ (College).

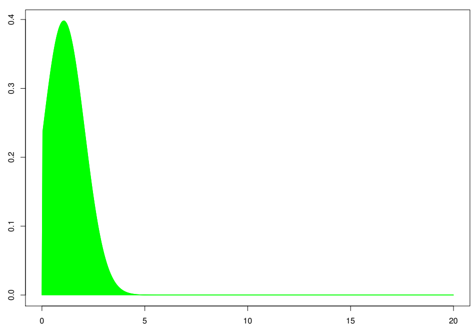
Figure 7a: Posterior Densities for the Elements of the Logistic Function (Age)



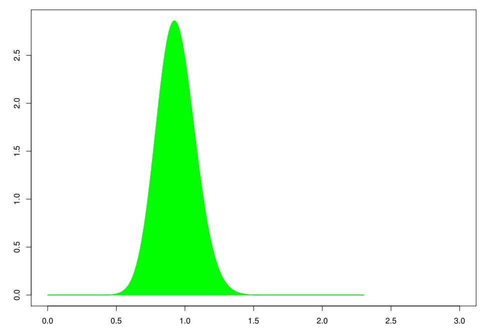
(g) Job-Tenure Threshold (All Heads).



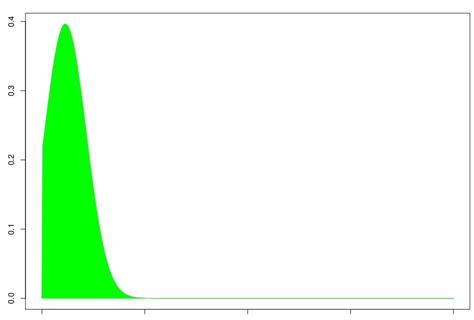
(h) Smoothing Parameter γ (All Heads).



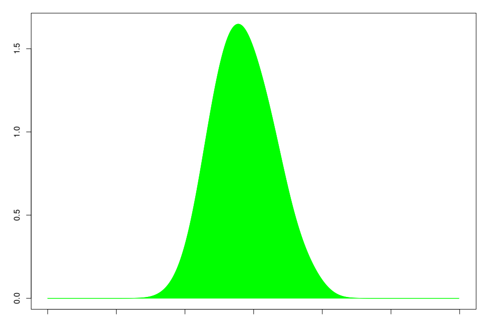
(i) Job-Tenure Threshold (High School).



(j) Smoothing Parameter γ (HS).

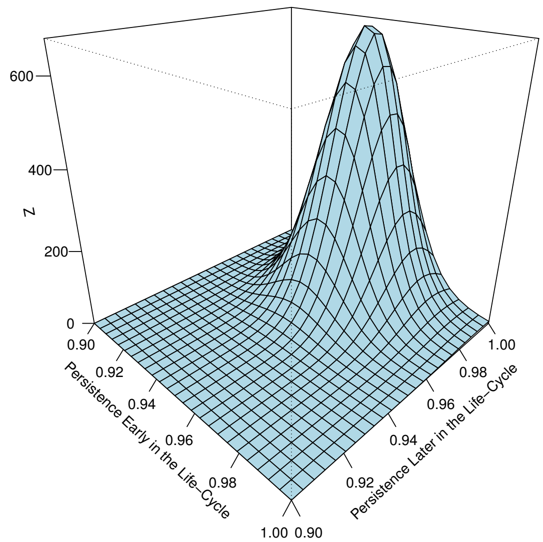


(k) Job-Tenure Threshold (College).

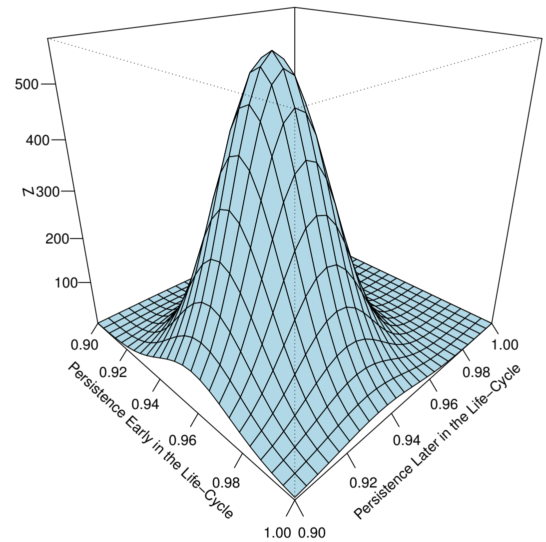


(l) Smoothing Parameter γ (College).

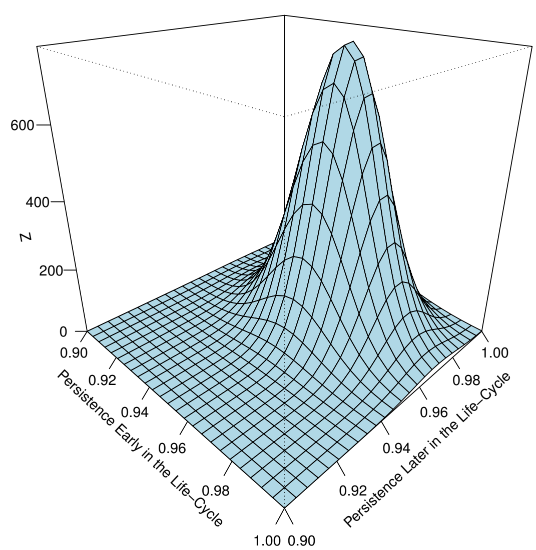
Figure 7b: Posterior Densities for the Elements of the Logistic Function (Job-Tenure)



(a) Heads with High-School Education

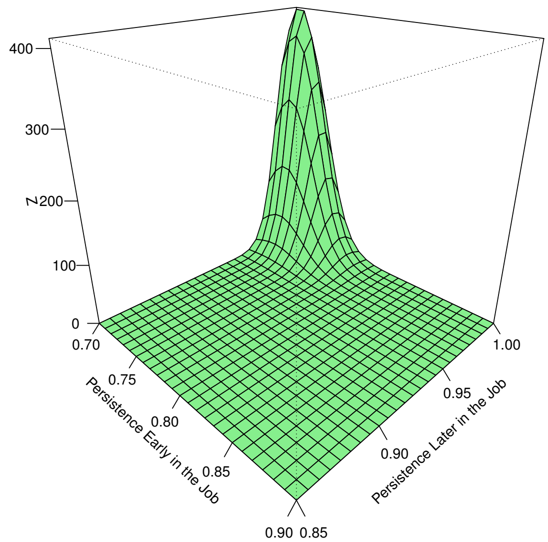


(b) Heads with College Education

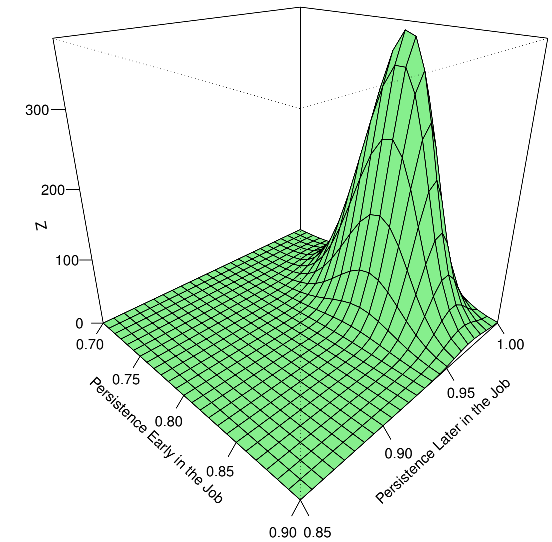


(c) All Heads of the Household

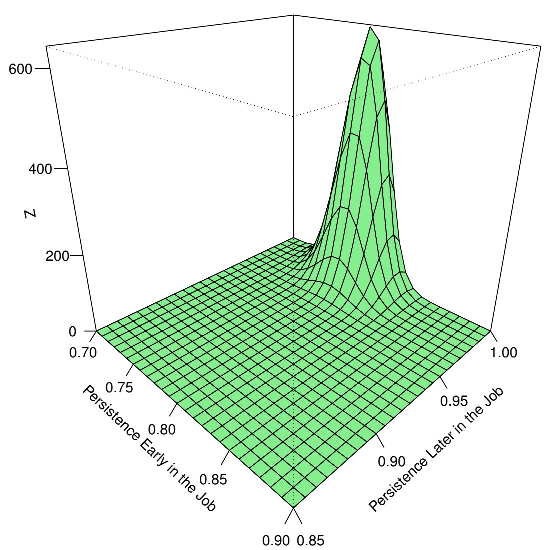
Figure 8a: Density Plots for ρ_1 and ρ_2 (Age).



(d) Heads with High-School Education

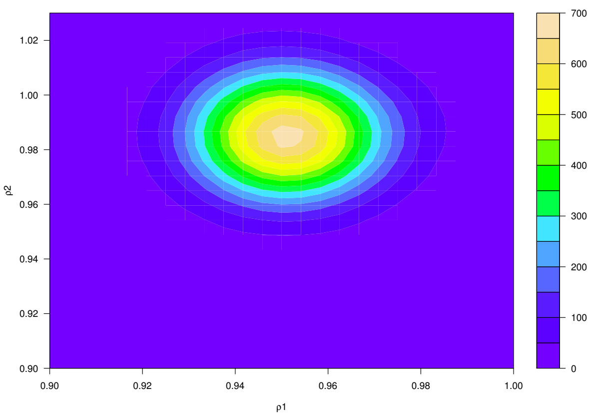


(e) Heads with College Education

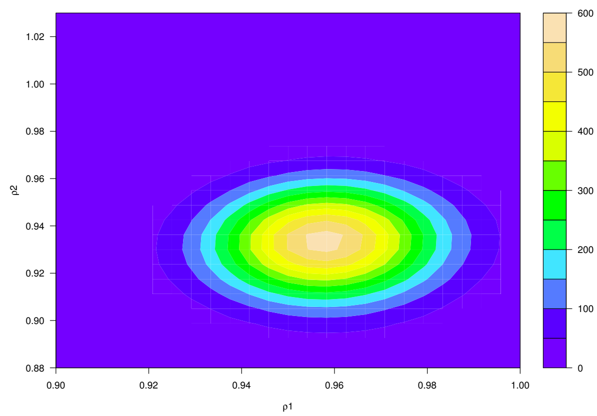


(f) All Heads of the Household

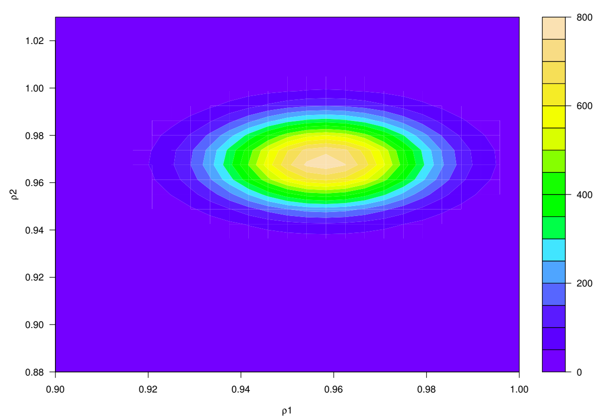
Figure 8b: Density Plots for ρ_1 and ρ_2 (Job-Tenure).



(a) Heads with High-School Education

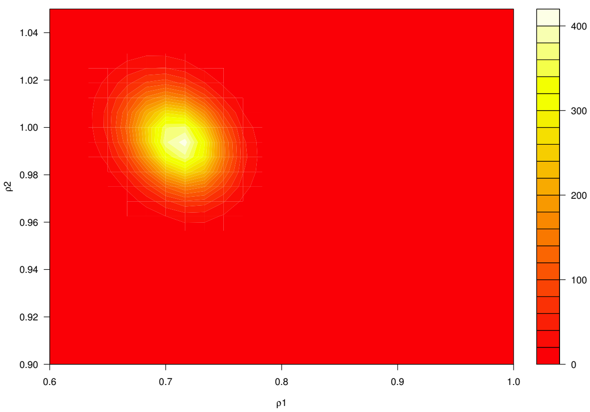


(b) Heads with College Education

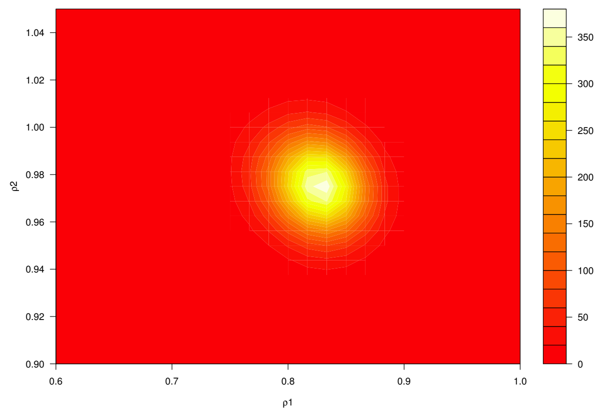


(c) All Heads of the Household

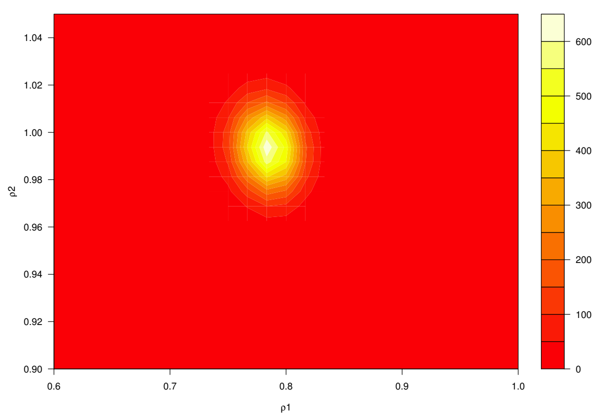
Figure 9a: Contour Plots for ρ_1 and ρ_2 (Age).



(d) Heads with High-School Education

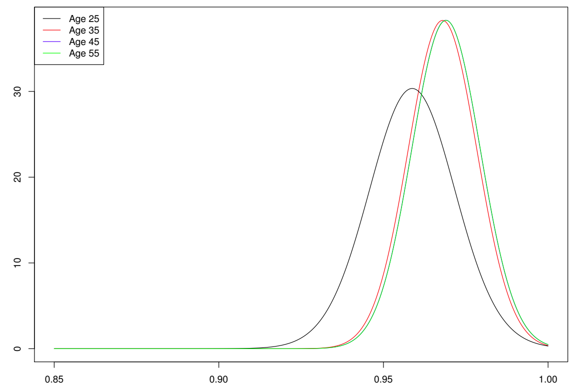


(e) Heads with College Education

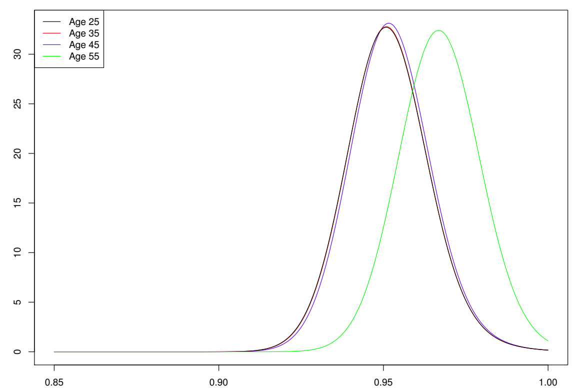


(f) All Heads of the Household

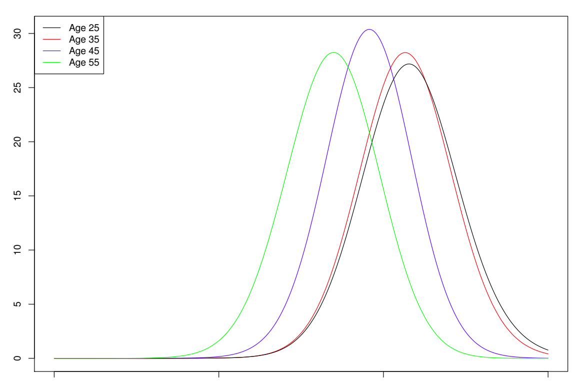
Figure 9b: Contour Plots for ρ_1 and ρ_2 (Job-Tenure).



(a) All Heads of the Household.

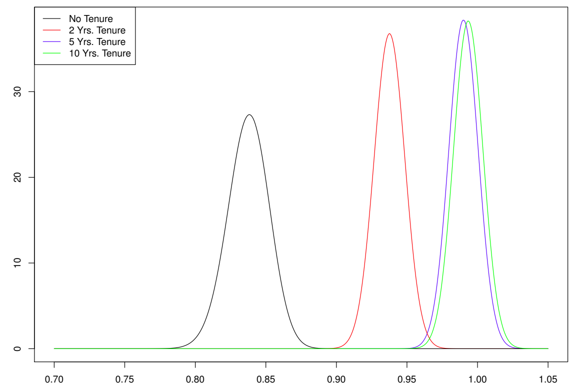


(b) Heads with High-School Education.

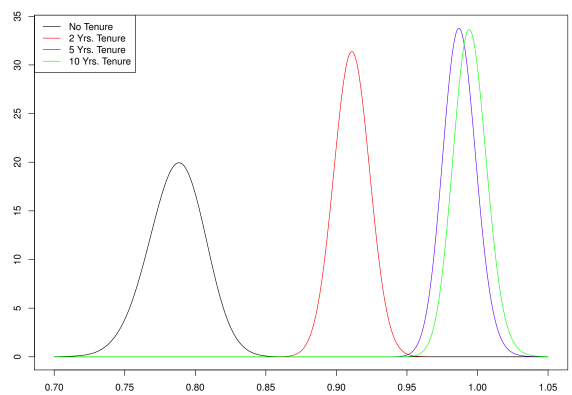


(c) Heads with College Education.

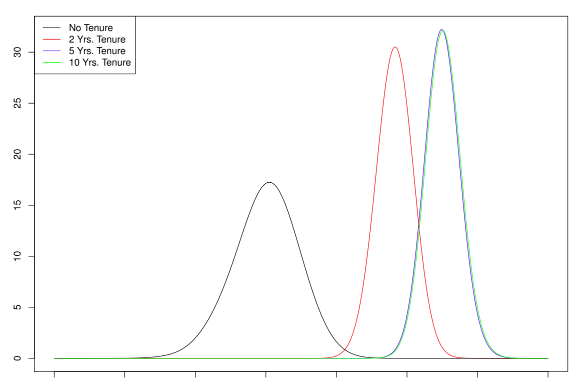
Figure 10a: Posterior Densities for the Correlation Coefficient ρ at different ages.



(d) All Heads of the Household.

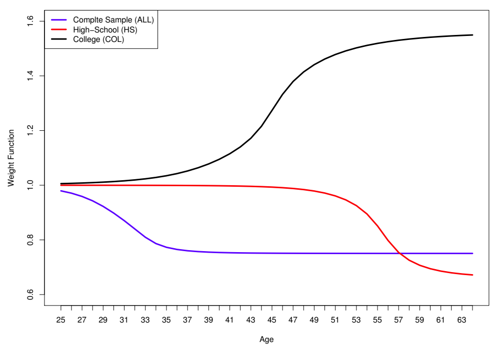


(e) Heads with High-School Education.

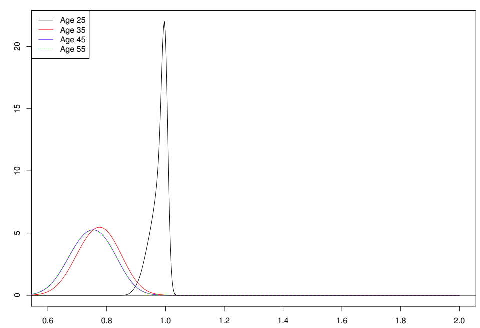


(f) Heads with College Education.

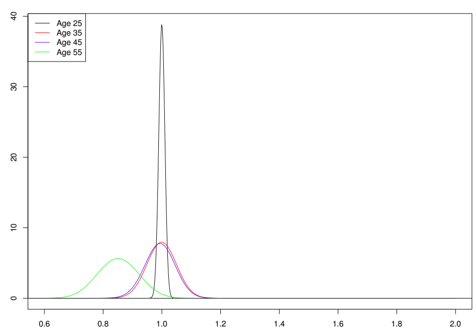
Figure 10b: Posterior Densities for the Correlation Coefficient ρ at different Job-Tenures.



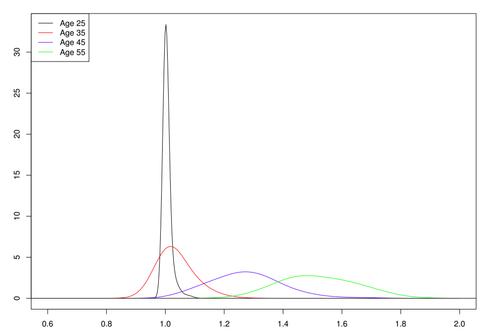
(a) Posterior Mean by Age



(b) Posterior Densities by Age (ALL)

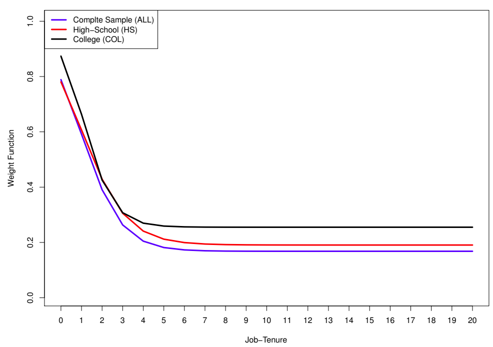


(c) Posterior Densities by Age (HS)

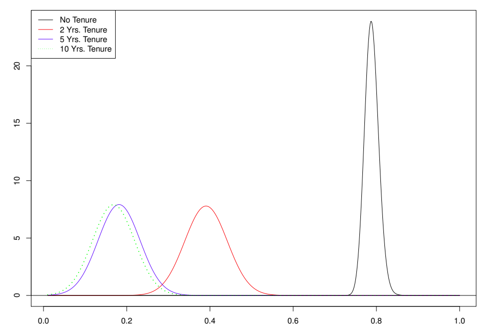


(d) Posterior Densities by Age (COL)

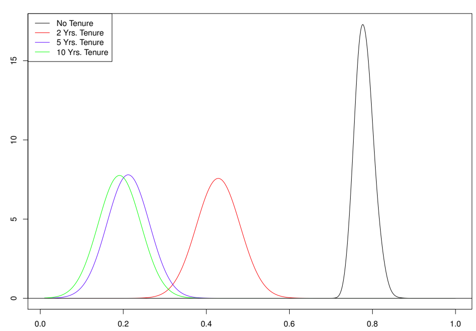
Figure 11a: Variance's Weight Function $k_t^i\,(\gamma, c, \phi)$.



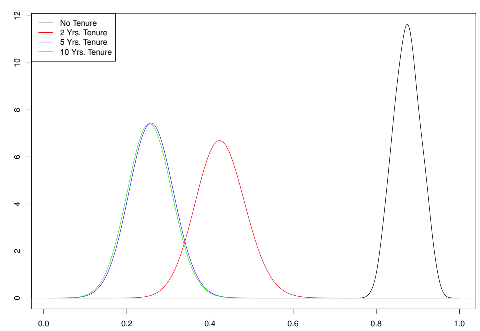
(e) Posterior Mean by Job-Tenure



(f) Posterior Densities by Tenure (ALL)

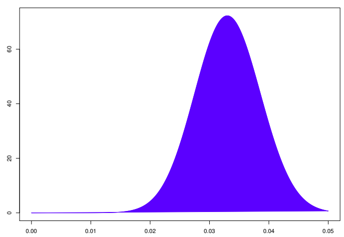


(g) Posterior Densities by Tenure (HS)

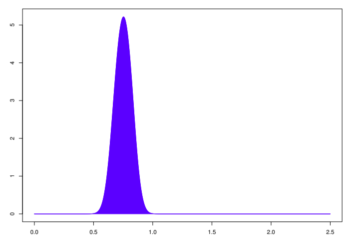


(h) Posterior Densities by Tenure (COL)

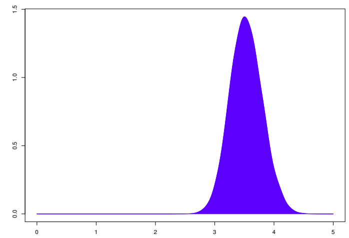
Figure 11b: Variance's Weight Function $k_t^i\,(\gamma, c, \phi)$.



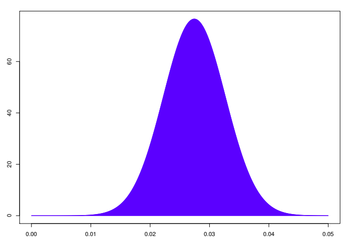
(a) Variance g_1^{-1} (ALL)



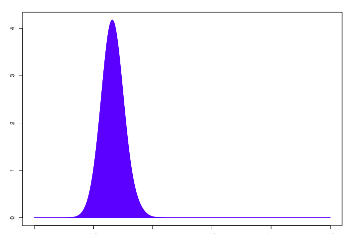
(b) Variances Ratio ϕ (ALL)



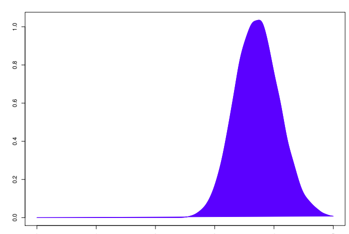
(c) Loading Factor λ^{-1} (ALL)



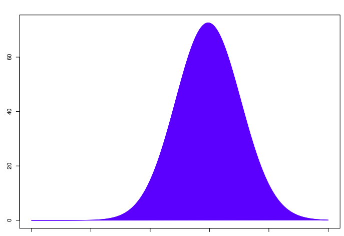
(d) Variance g_1^{-1} (HS)



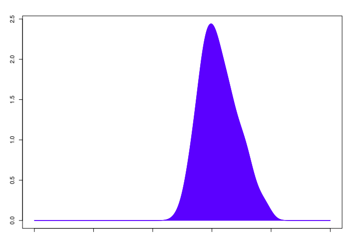
(e) Variances Ratio ϕ (HS)



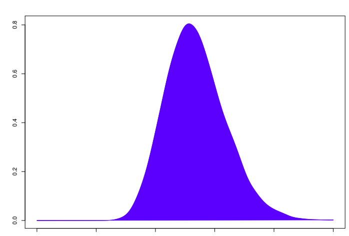
(f) Loading Factor λ^{-1} (HS)



(g) Variance g_1^{-1} (COL)

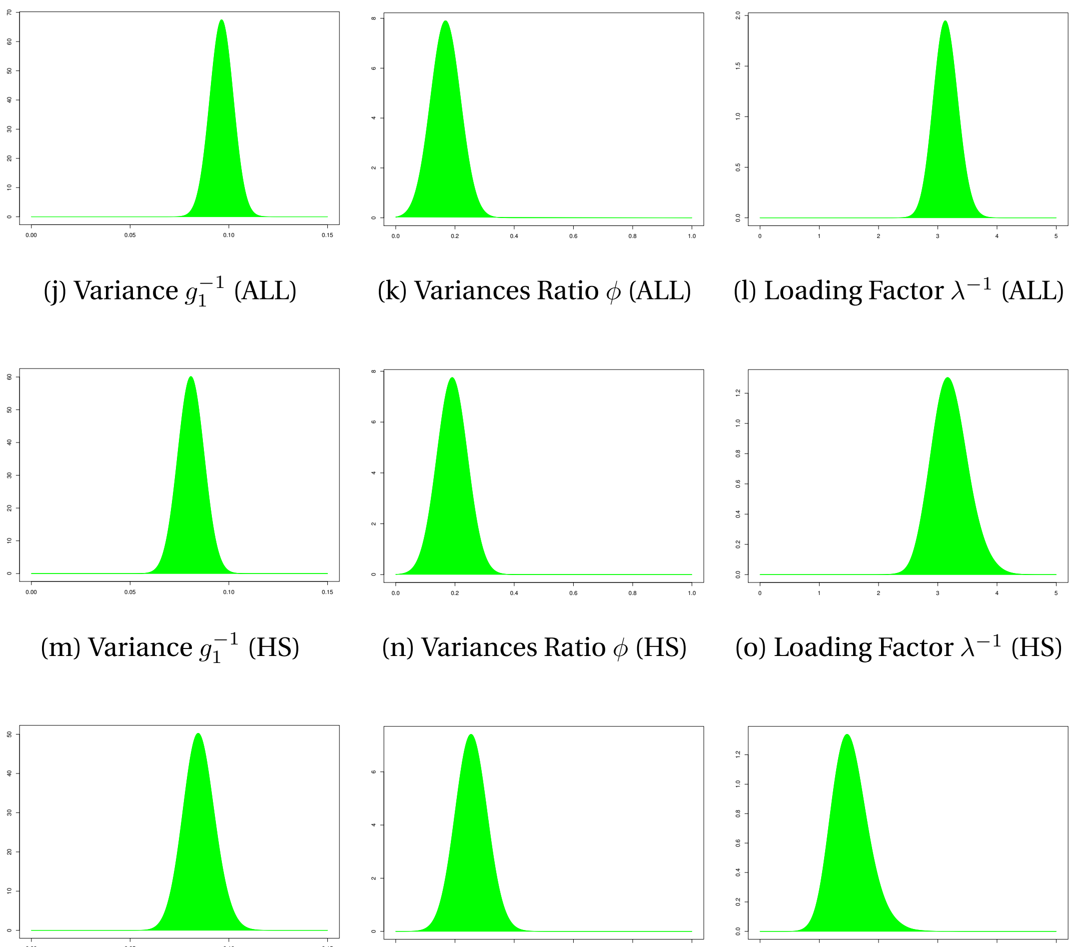


(h) Variances Ratio ϕ (COL)



(i) Loading Factor λ^{-1} (COL)

Figure 12a: Variance Components' Posterior Densities (2-Regimes Model; Age).

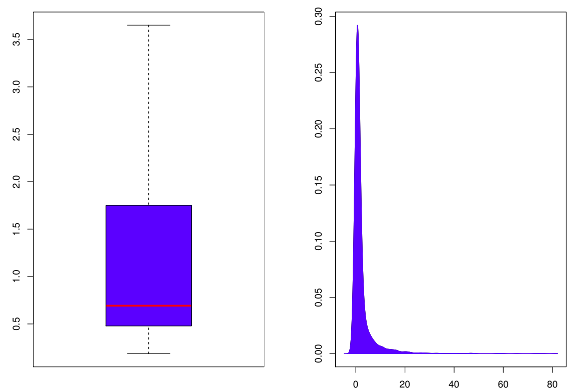


(j) Variance g_1^{-1} (ALL) (k) Variances Ratio ϕ (ALL) (l) Loading Factor λ^{-1} (ALL)

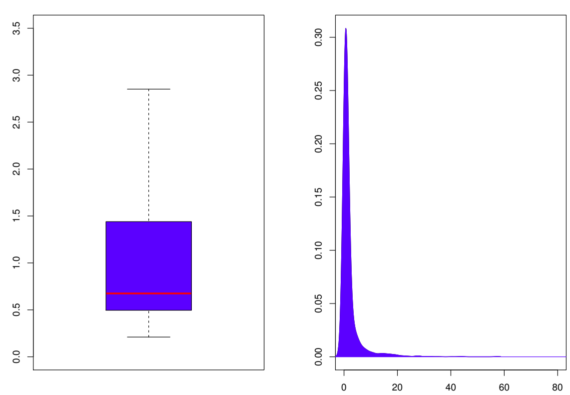
(m) Variance g_1^{-1} (HS) (n) Variances Ratio ϕ (HS) (o) Loading Factor λ^{-1} (HS)

(p) Variance g_1^{-1} (COL) (q) Variances Ratio ϕ (COL) (r) Loading Factor λ^{-1} (COL)

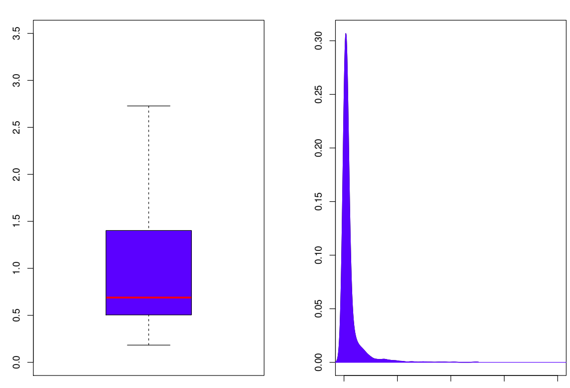
Figure 12b: Variance Components' Posterior Densities (2-Regimes Model; Job-Tenure).



(a) All Heads of the Household.

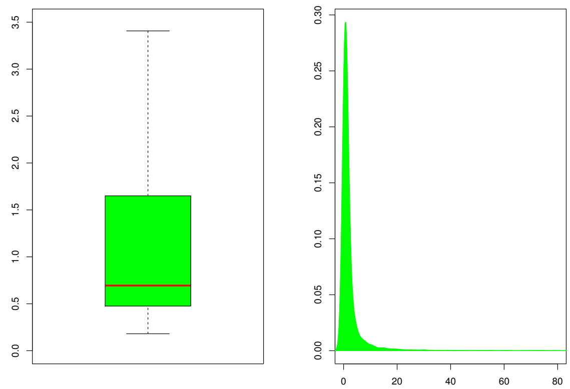


(b) Heads with High-School Education.

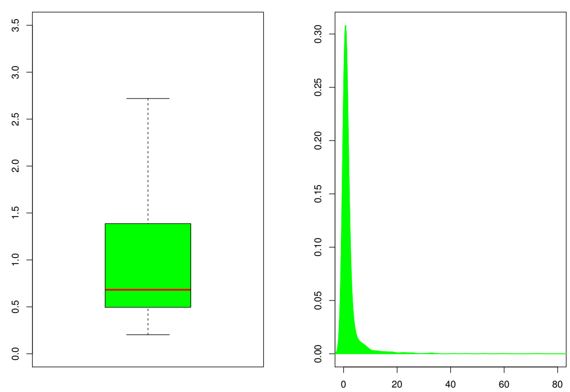


(c) Heads with College Education.

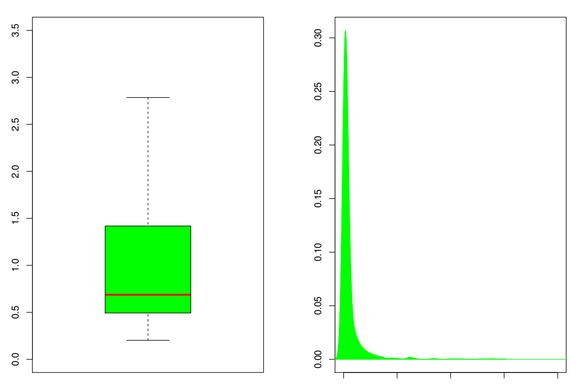
Figure 13a: Individual-Specific Loading Factor κ_i^{-1} Posterior Distribution (2-Regimes Model; Age).



(d) All Heads of the Household.



(e) Heads with High-School Education.



(f) Heads with College Education.

Figure 13b: Individual-Specific Loading Factor κ_i^{-1} Posterior Distribution (2-Regimes Model; Job-Tenure).